Bayes-Adaptive Planning for Data-Efficient Verification of Uncertain Markov Decision Processes

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Outline

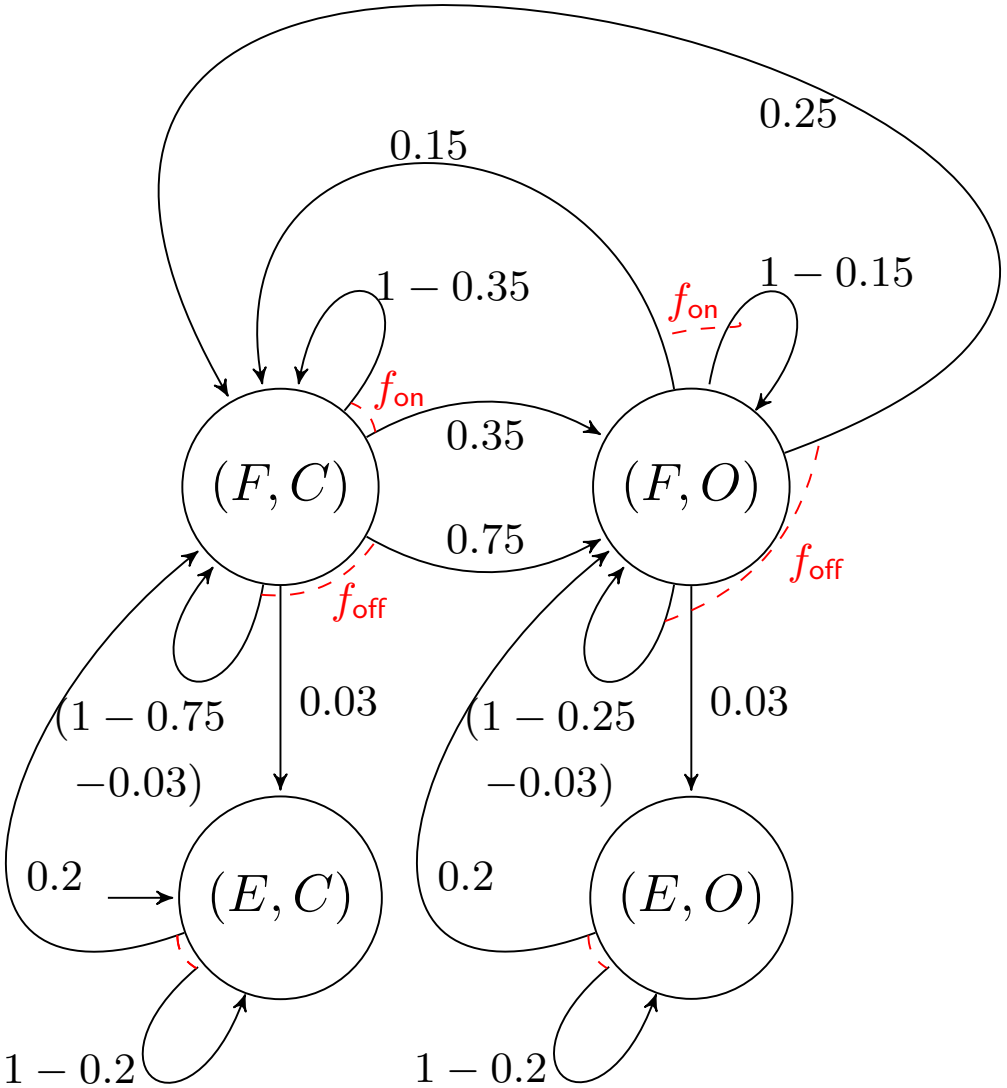
Automated Formal Verification - New Frontiers

- Verification of physical systems
  - Need to reduce the gap between data and models
  - Principled integration of learning and verification

Figure: Advanced modelling for smart buildings: Building automation system setup in rooms 478/9 at Oxford CS with Estate Services at Oxford, Honeywell (CZ), Trend Control (UK)

Applications: control of temperature, humidity, CO_2, model-based predictive maintenance of devices, fault-tolerant certified control, demand-response over smart grids

Discrete-time Markov Decision Process (MDP)



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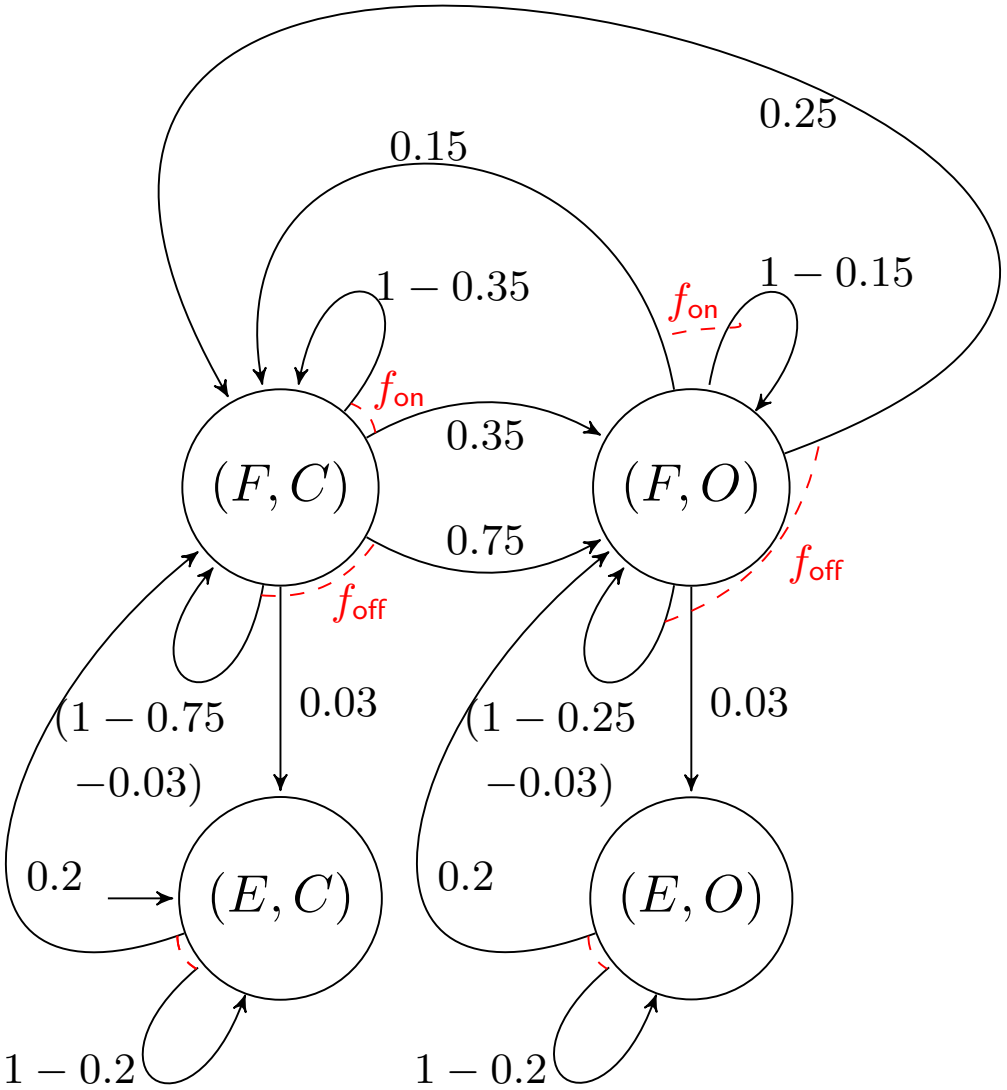


transition probability function

finite set of actions

finite set of initial states

finite set of states

$$\mathcal{M} = (S, A, \mathcal{T}, \iota)$$

Windows are open (O) or closed (C)
Zone is occupied (F) or not (E)

$$\mathcal{T} \colon S \times A \times S \to [0, 1]$$

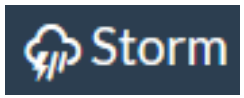
$$\mathcal{T} = \left[\begin{array}{cccc} 0.8 & 0.2 & 0 & 0 \\ \hline 0 & 0.65 & 0.35 & 0 \\ 0.03 & 0.22 & 0.75 & 0 \\ \hline 0 & 0.15 & 0.85 & 0 \\ 0 & 0.25 & 0.72 & 0.03 \\ \hline 0 & 0 & 0.2 & 0.8 \end{array}\right]$$

Probabilistic Computational Tree Logic (PCTL) Properties

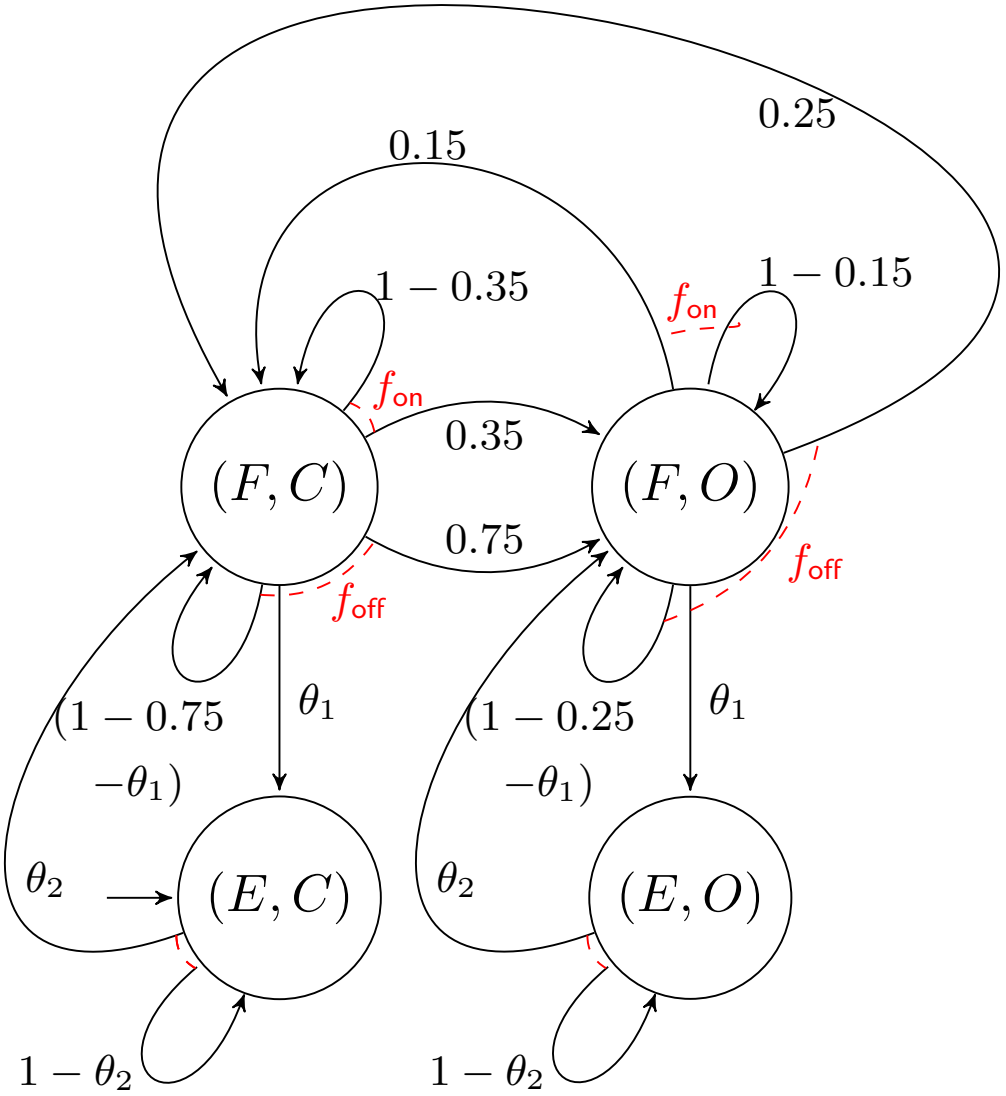
$non - nested$ PCTL properties compatible with Storm

$\mathbf{P}_{\geq 0.35} \neg (\text{true}\ \mathcal{U}^{\leq 20}\ (\text{E}, \text{O}))$

$\mathbf{P}_{\geq 0.5} (\text{true}\ \mathcal{U}\ (\text{E}, \text{O}))$



Uncertain Markov Decision Process



Parametric Markov Decision Process (pMDP)

finite set of states — finite set of actions — finite set of initial states

$$\mathcal{M}_p = (S, A, \mathcal{T}_p, \iota, \Theta)$$

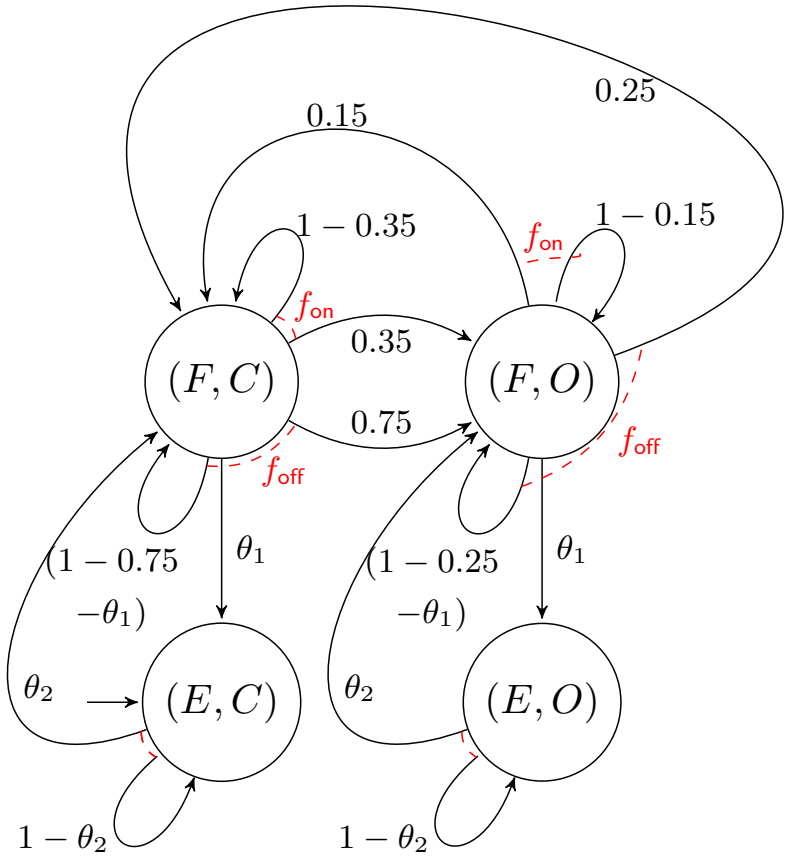
$$\mathcal{T}_p = \left[\begin{array}{cccc} 1-\theta_2 & \theta_2 & 0 & 0 \\ \hline 0 & 0.65 & 0.35 & 0 \\ \theta_1 & 1-0.75-\theta_1 & 0.75 & 0 \\ \hline 0 & 0.15 & 0.85 & 0 \\ 0 & 0.25 & 1-0.25-\theta_1 & \theta_1 \\ \hline 0 & 0 & \theta_2 & 1-\theta_2 \end{array}\right]$$

$$\Theta \subseteq [0,1]^n = [0, 0.25] \times [0, 1]$$

Parameter synthesis: find values for parameters $\theta \in \Theta$ s.t. induced model $\mathcal{M}(\theta)$ satisfies φ:

$$\Theta_\varphi = \{\theta \in \Theta : M(\theta) \models \varphi\} \subseteq \Theta$$

We call Θ_φ the *feasible set*.

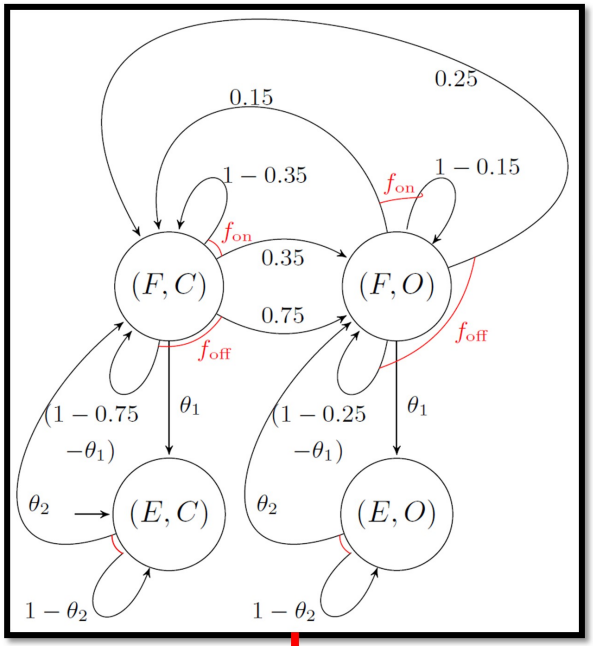


Parameter Synthesis

PCTL

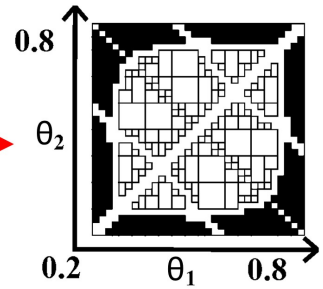
$\varphi = P_{>=0.5}[\text{true } \mathcal{U} \text{ (E,O)}]$

System



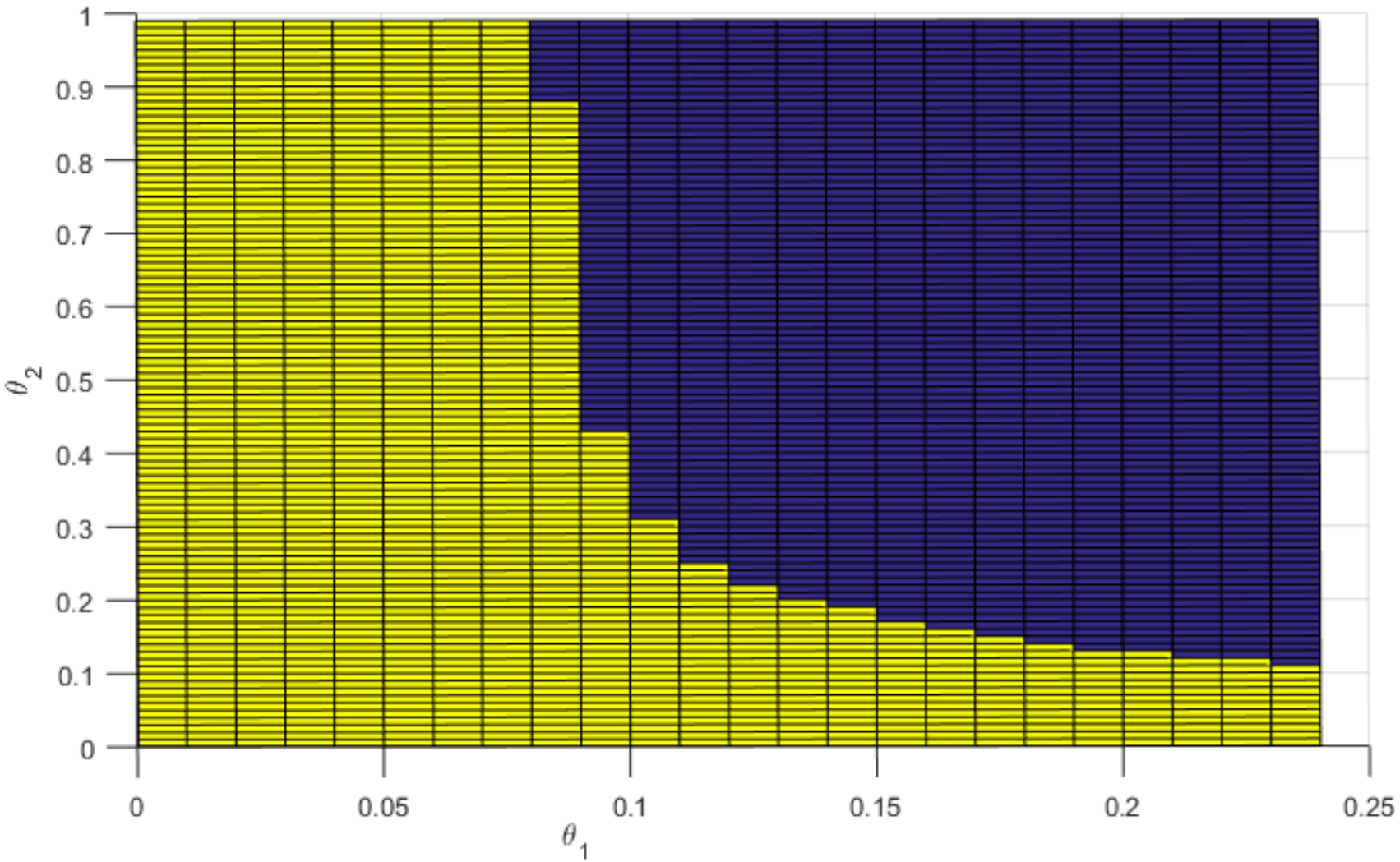
pMDP

Probabilistic Model Checker



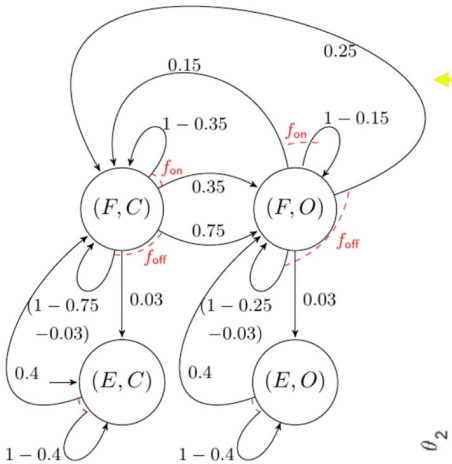
Regions ➔ Truth Values

Parameter Synthesis



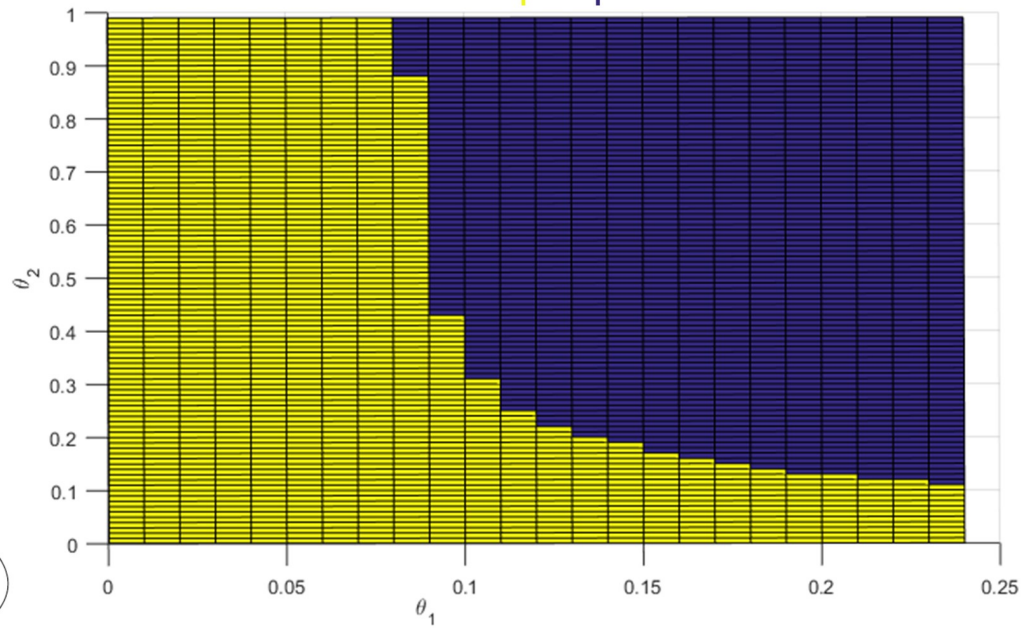
For the property $\varphi = \mathbf{P}_{>0.3}(\Box^{\leq 20}\ \neg(\mathsf{E}, \mathsf{O}))$, synthesis yields the feasible set Θ_φ (yellow set) which contains all valuations of parameters that satisfy φ

Model Classification



$S \models \varphi$

$S \not\models \varphi$



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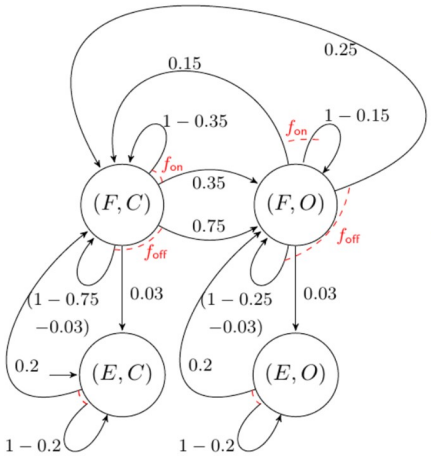
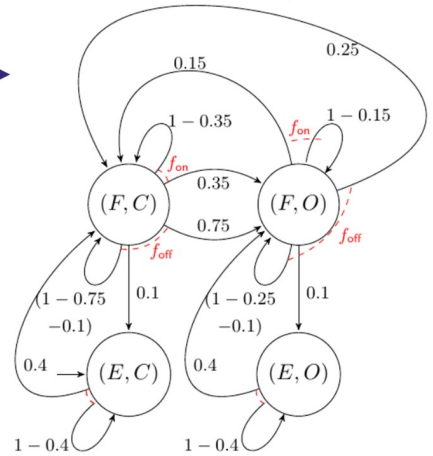
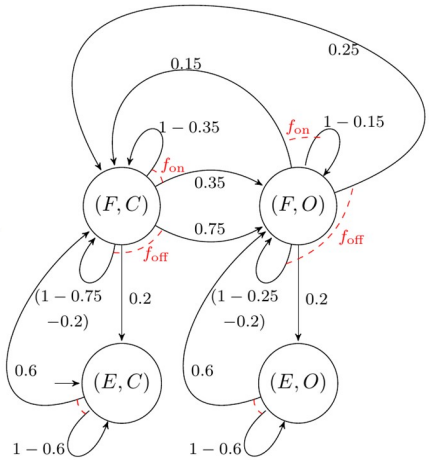






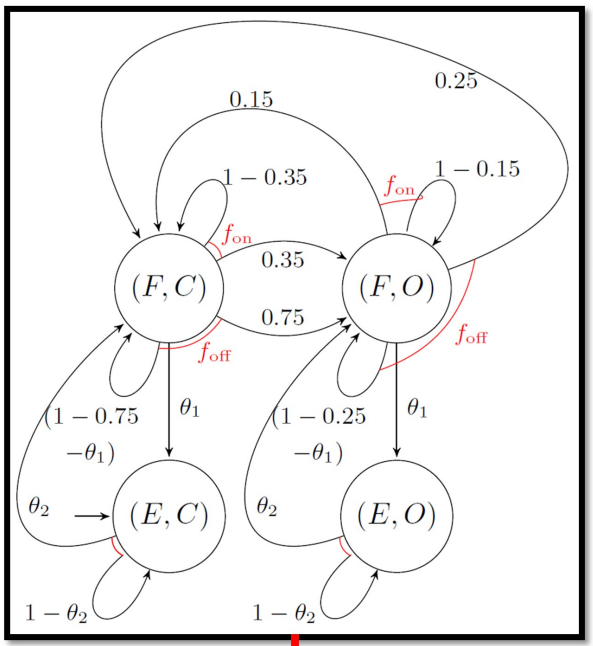
System Verification

Does the system satisfy the property?

PCTL

$\varphi = P_{>=0.5}[\text{true } \mathcal{U} \text{ (E,O)}]$

System



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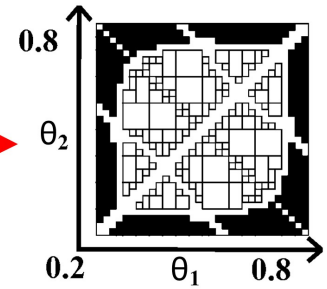
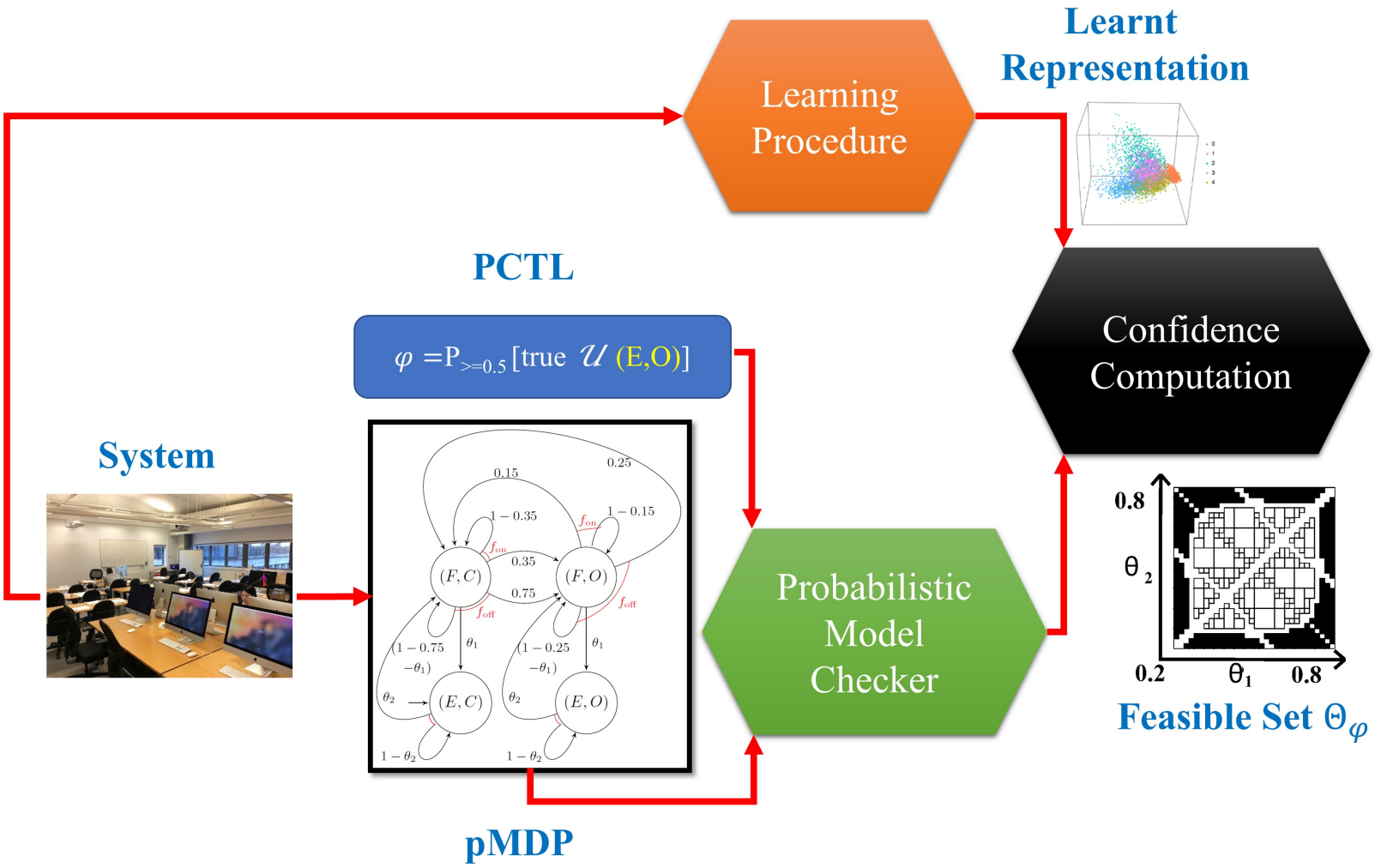


Regions ➔ Truth Values

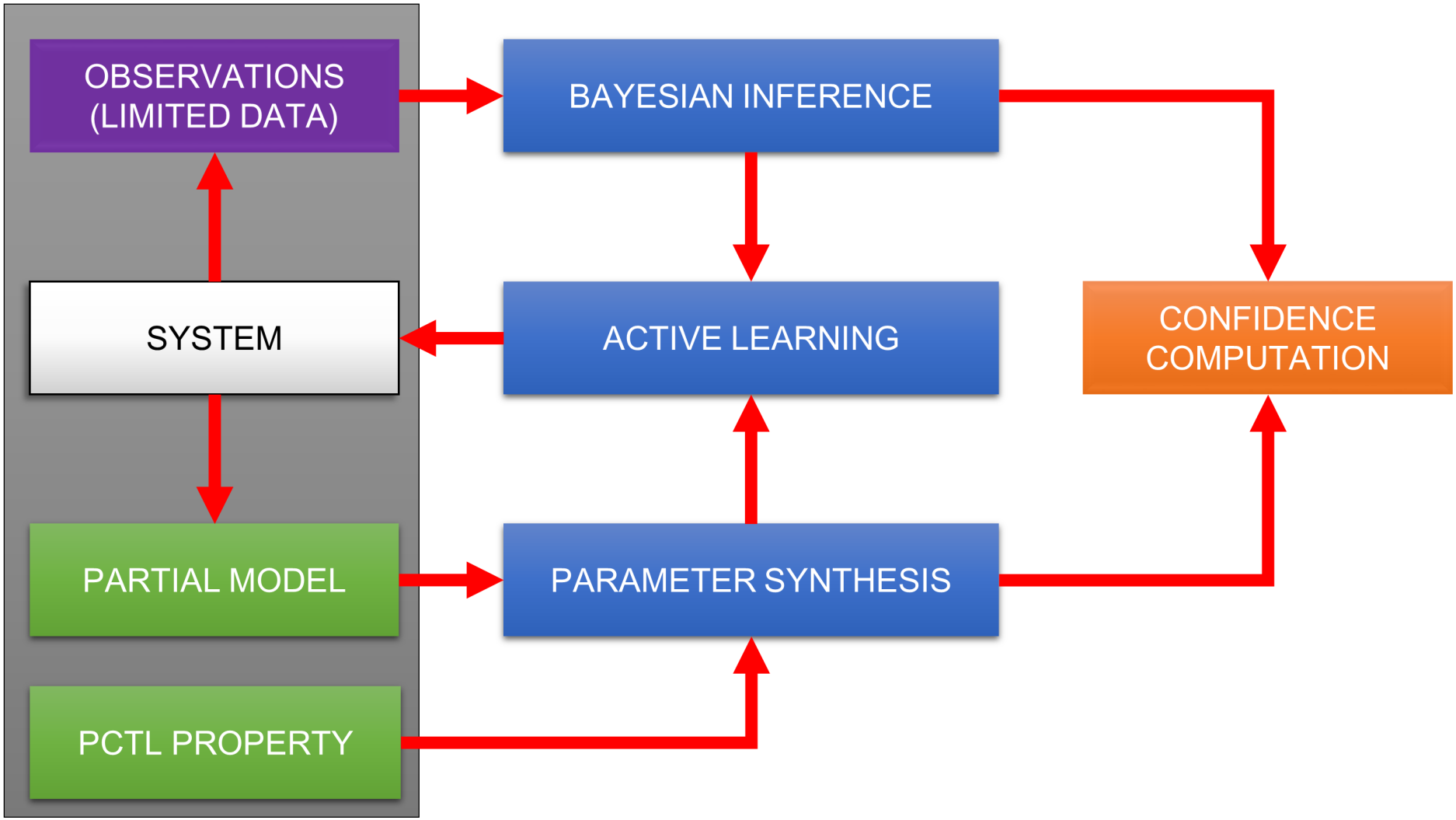
Outline

Integrated Learning and Verification Framework

Incomplete knowledge in pMDP can complement the learning process



Overview


OBSERVATIONS
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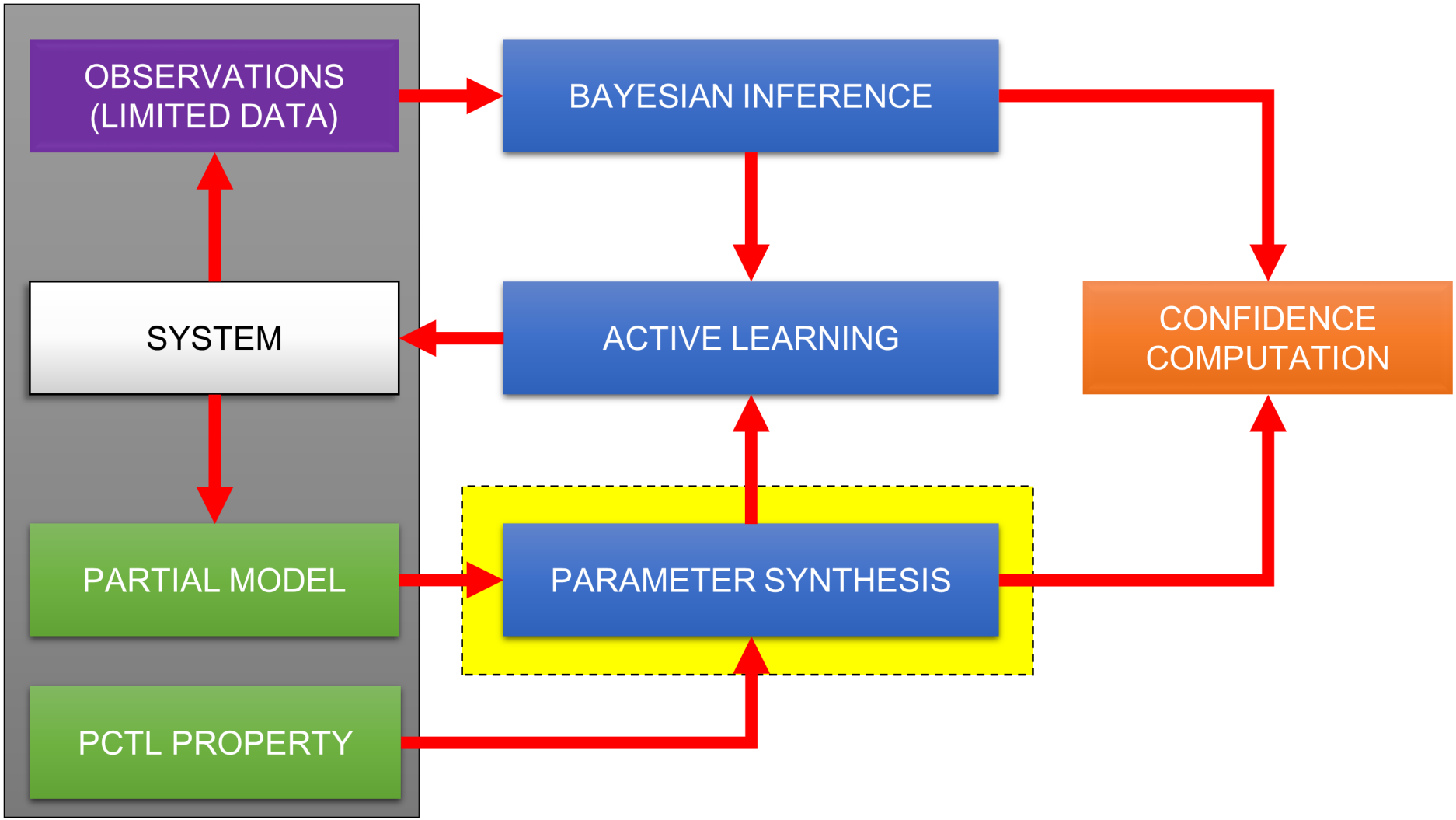
Overview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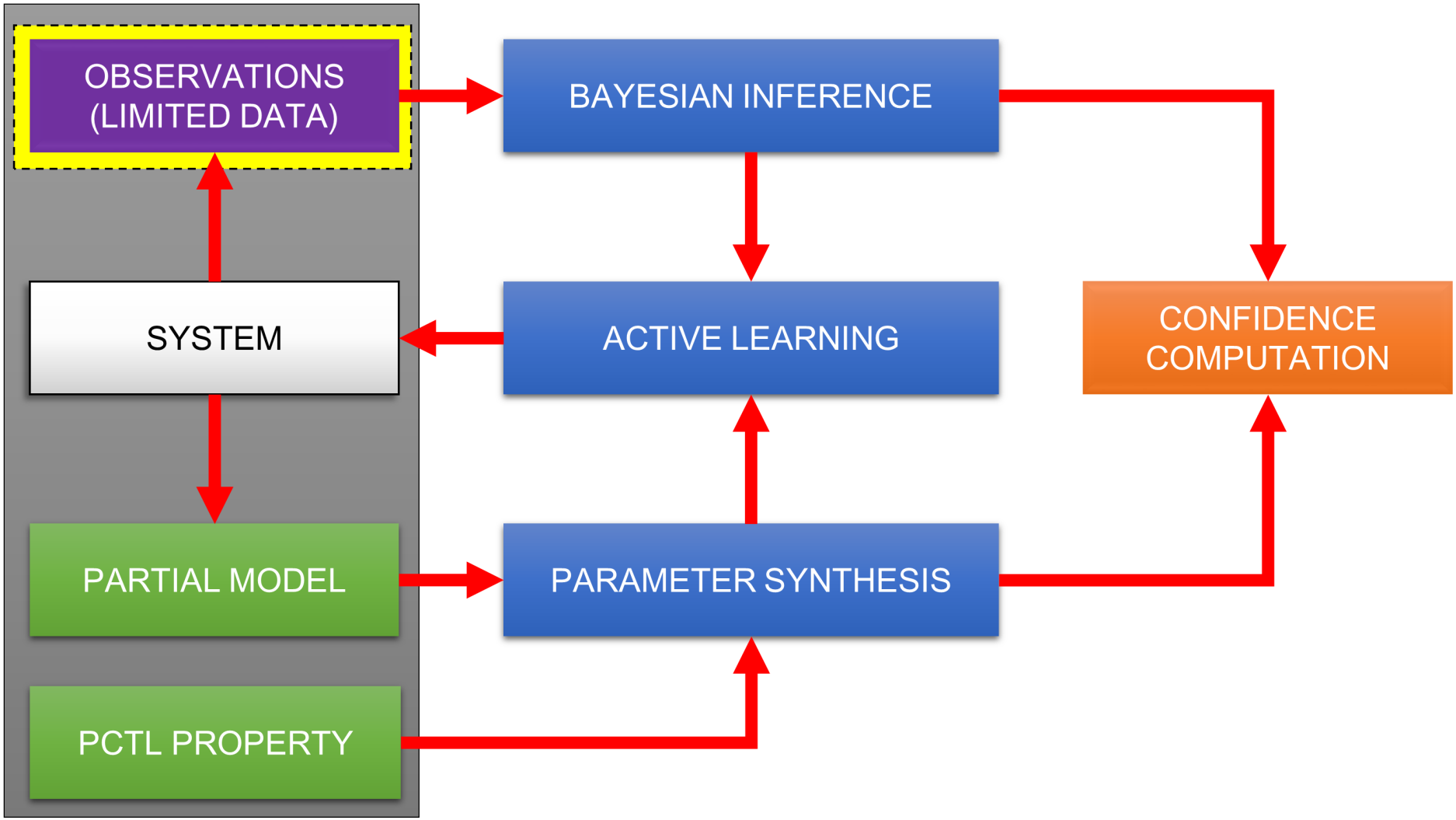
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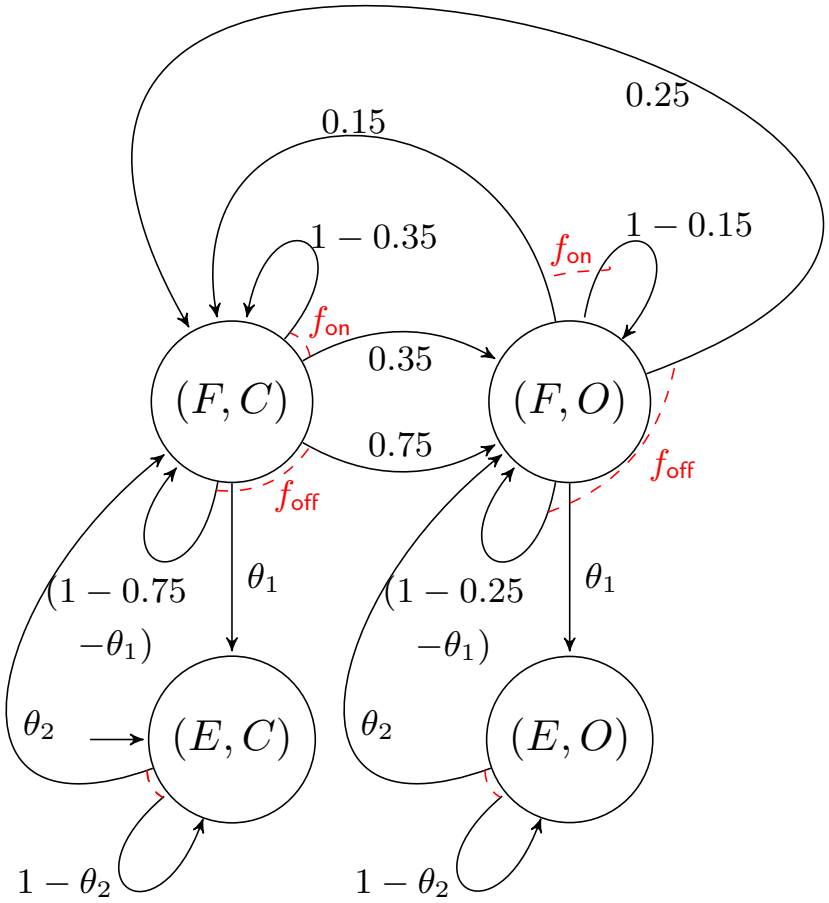
Observations/Data



Group transition counts by parameter for $\exists \bar{s} \in S : \mathcal{T}_p(s,a,\bar{s}) = \theta_i$

$$C(\theta_i) = \sum C^a_{s,s'} \text{ for } \mathcal{T}_p(s,a,s') = \theta_i$$

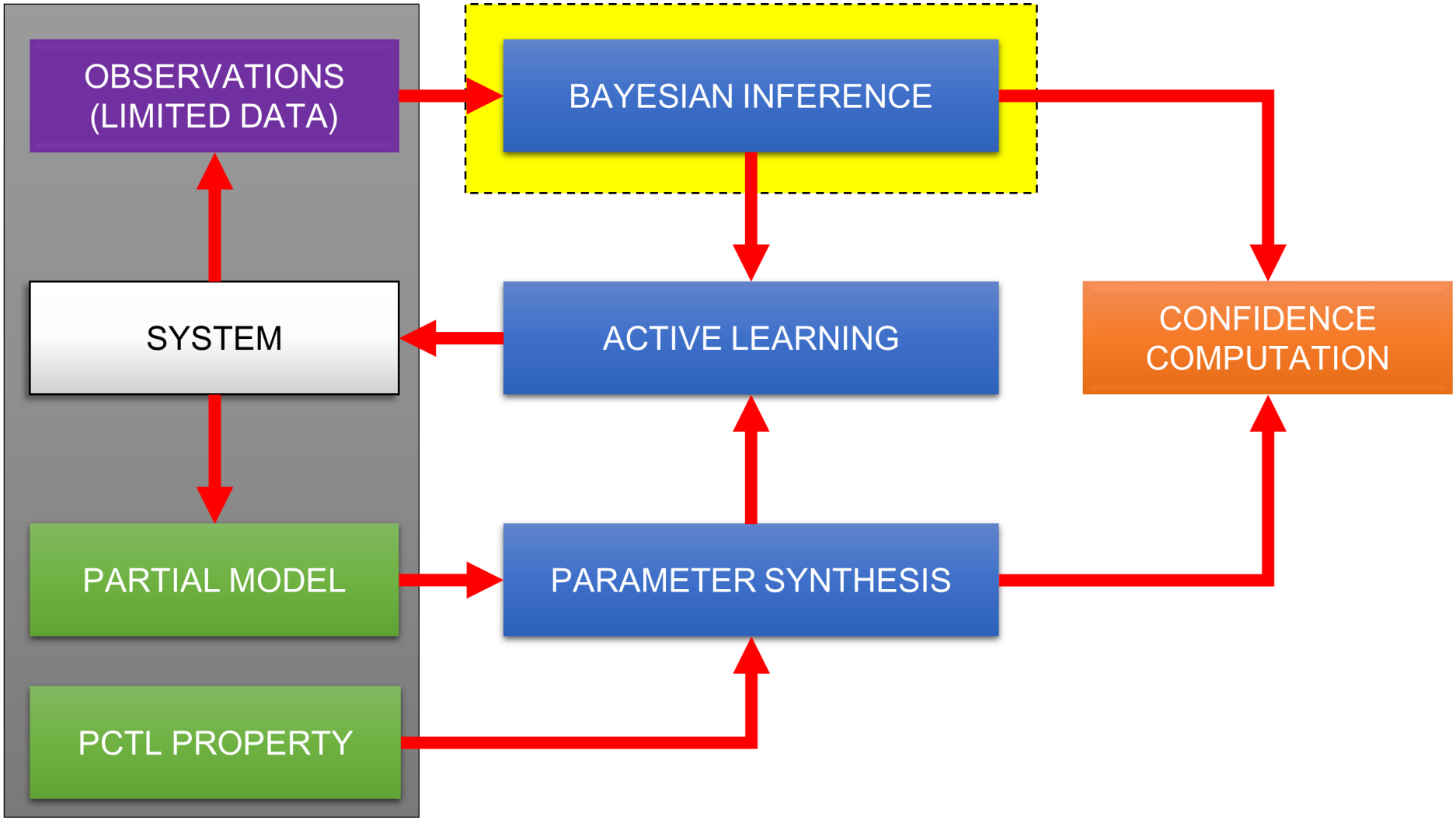
$$C'(\theta_i) = \sum C^a_{s,s'} \text{ for } \mathcal{T}_p(s,a,s') \neq \theta_i$$

$$\bar{C}(\theta_i) = C(\theta_i), C'(\theta_i)$$

Trace/Trajectory :

$$(\{E,C\},\cdot) \xrightarrow{\theta_2} (\{F,C\},f_{\text{off}}) \xrightarrow{1-0.75-\theta_1} (\{F,C\},f_{\text{off}}) \xrightarrow{0.75} (\{F,O\},f_{\text{off}}) \xrightarrow{0.25} (\{F,C\},f_{\text{off}}) \xrightarrow{\theta_1} (\{E,C\},\cdot) \xrightarrow{1-\theta_2} (\{E,C\},\cdot)\ldots$$

Overview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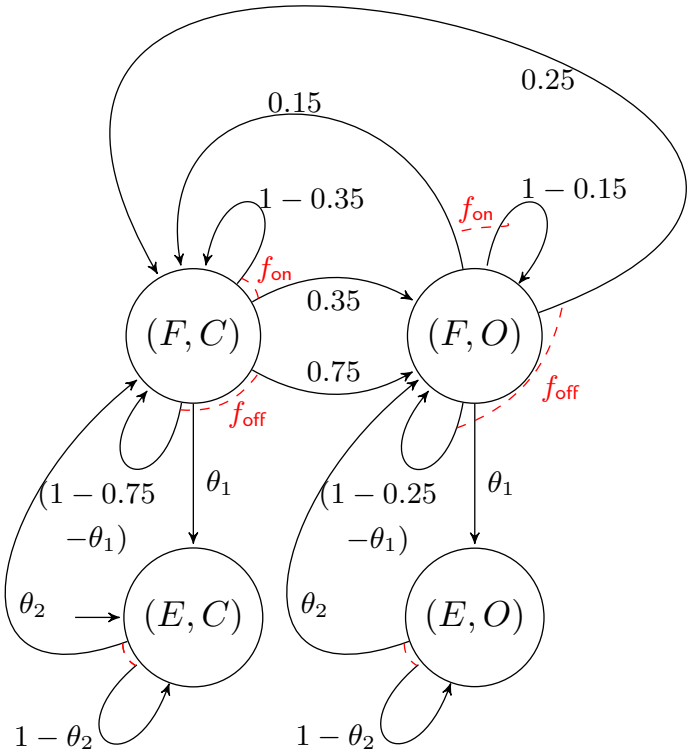
Bayesian Inference



$$P(\theta_i \mid \mathscr{D}) = \frac{P(\mathscr{D} \mid \theta_i)P(\theta_i)}{P(\mathscr{D})}$$

observed — prior — binomial

$$= \frac{P(\theta_i)\theta_i^{C(\theta_i)}(1-\theta_i)^{C'(\theta_i)}}{P(\bar{C}(\theta_i))}$$

Select a conjugate prior - Beta distribution:

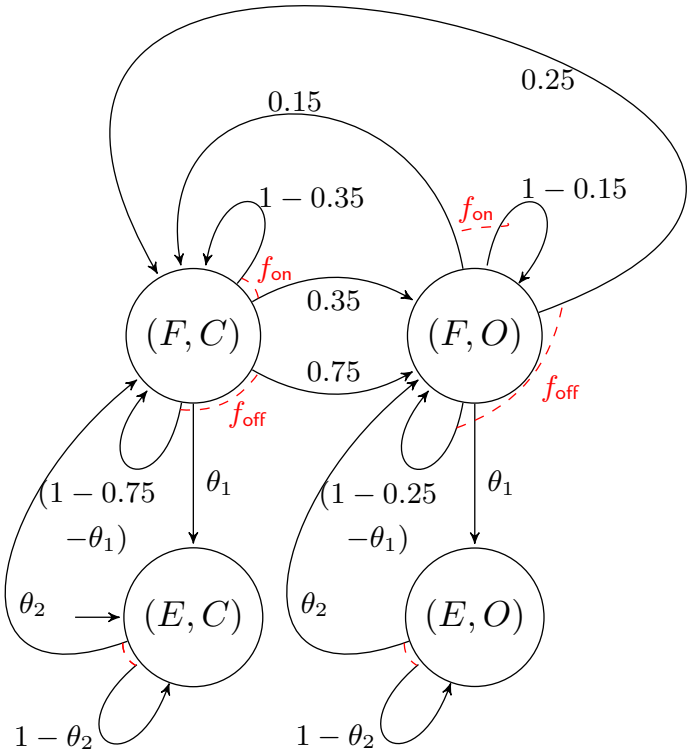
$$P(\theta_i) = Beta(\theta_i;\omega) \propto \theta_i^{\alpha-1}(1-\theta_i)^{\beta-1}$$

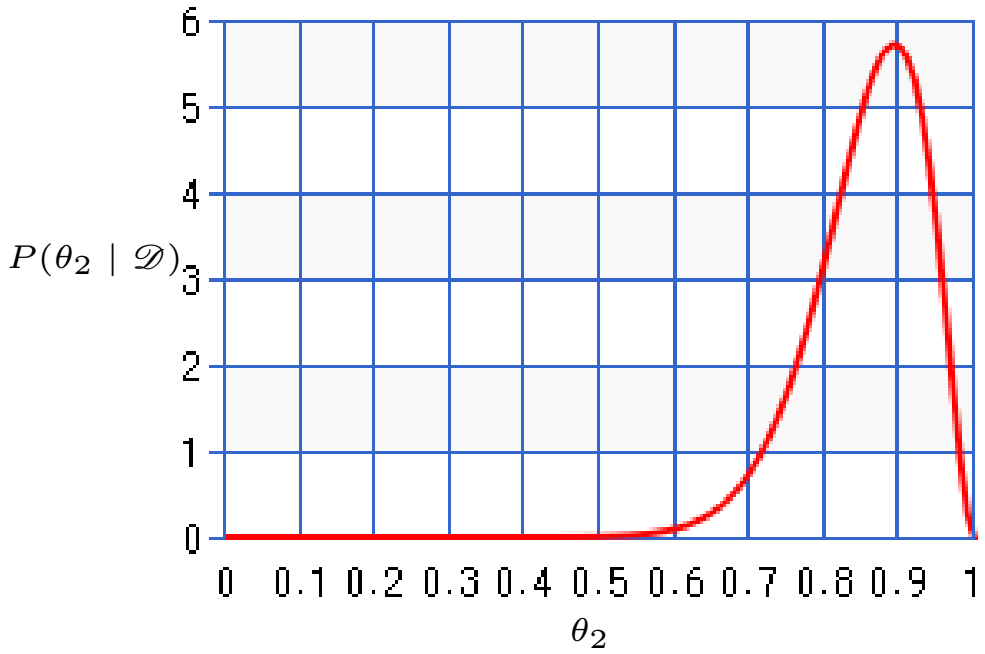
for pair $(\theta_i, 1-\theta_i)$, with $\omega = (\alpha, \beta)$ hyperparameters

Under Beta prior, posterior update is analytic:

$$P(\theta_i \mid \mathscr{D}) = Beta(\theta_i; \bar{C}(\theta_i) + \omega) \propto \theta_i^{C(\theta_i)}(1-\theta_i)^{C'(\theta_i)}$$

Bayesian Inference





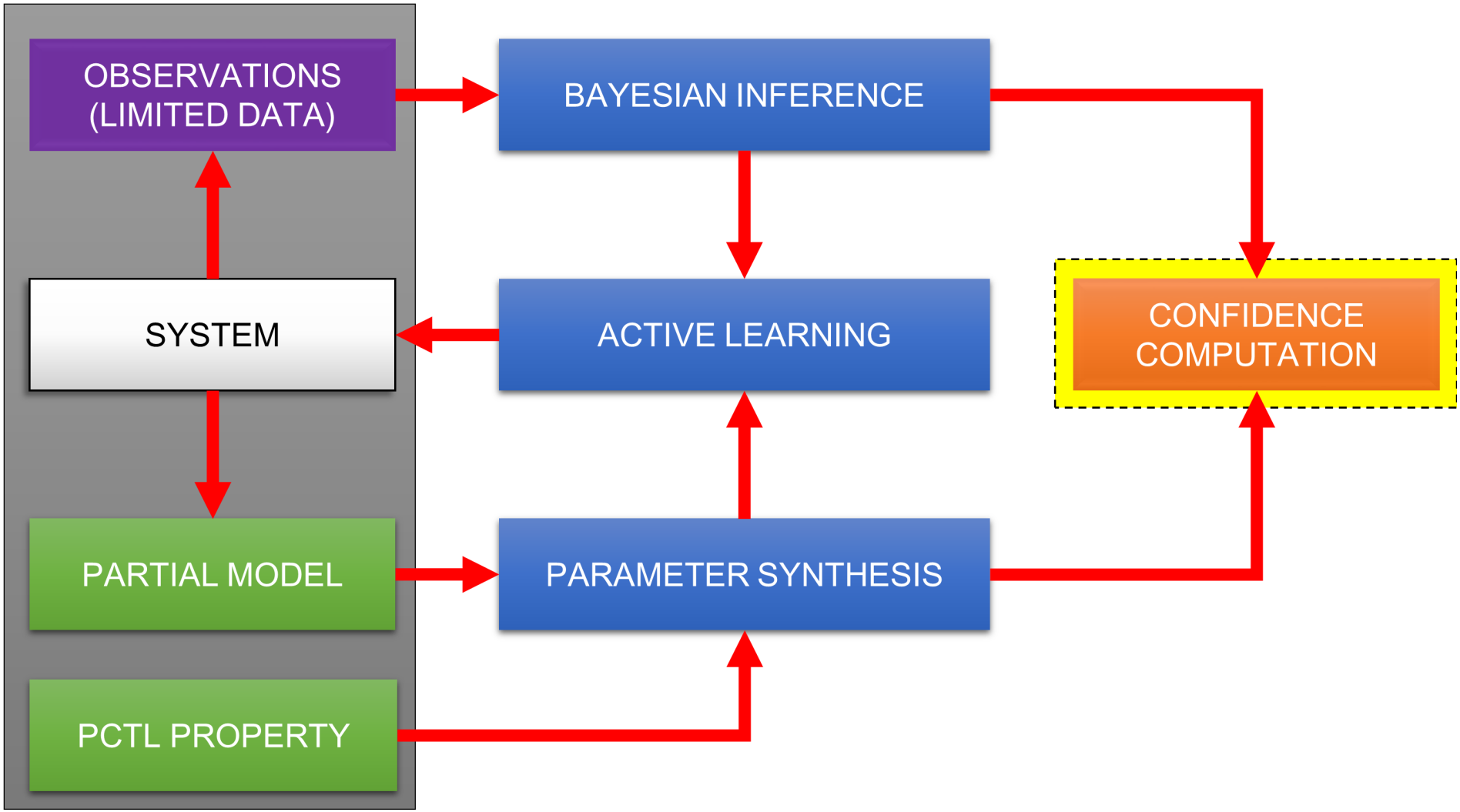
With prior $\alpha = (1, 1)$

$P(\theta_2 \mid \mathscr{D}) = Beta(\theta_2; 18, 3)$

Consider (E, C) and (E, O)

- Observe $\theta_2 \rightarrow C(\theta_2) = 17$
- Observe $(1 - \theta_2) \rightarrow C'(\theta_2) = 2$

Overview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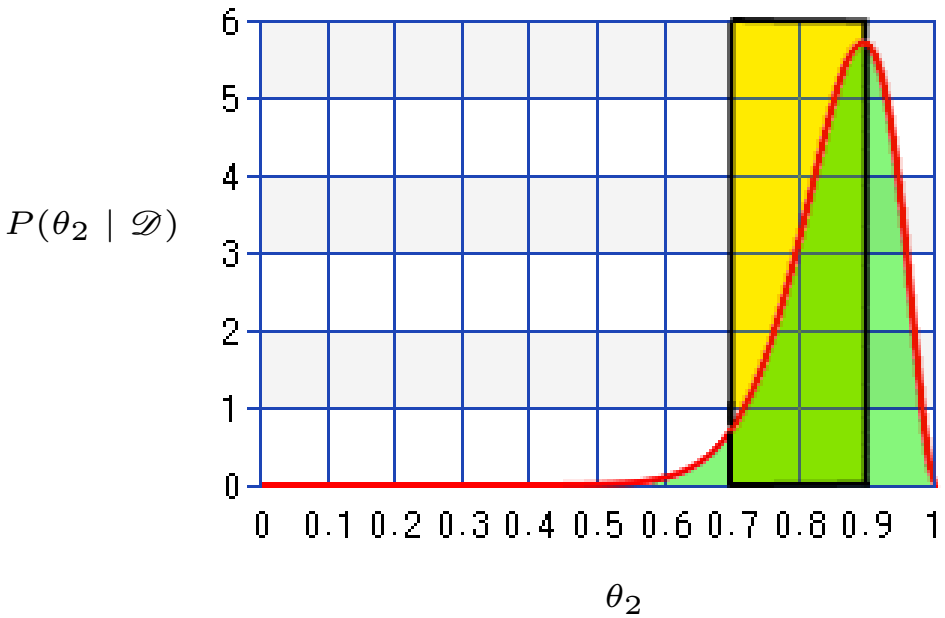
Confidence Computation

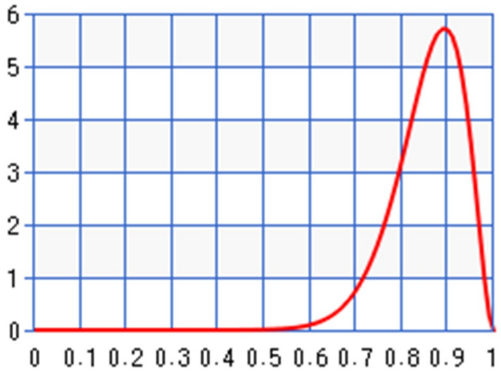
Confidence $\mathscr{C}$ on whether the system S satisfies the property φ:

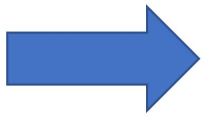
$$\mathscr{C} = \mathbb{P}(\mathsf{S} \models \varphi \mid \mathscr{D}) = \int_{\Theta_\varphi} P(\theta \mid \mathscr{D}) d\theta$$



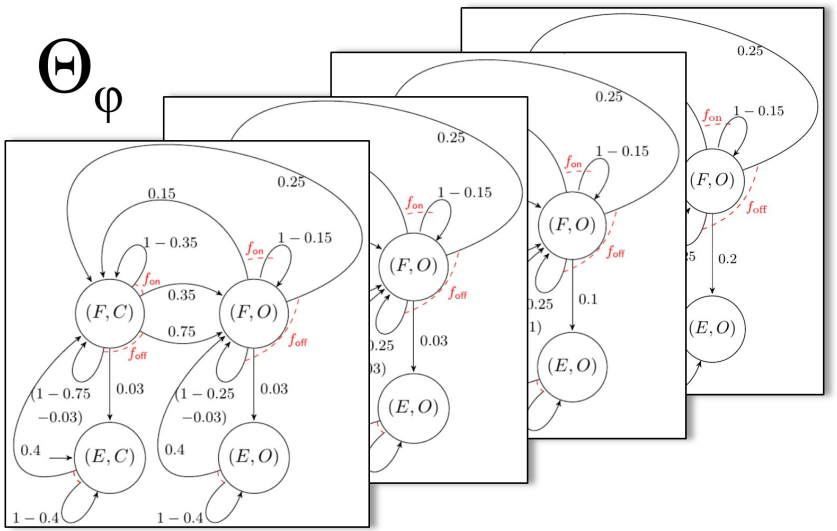
Confidence Computation

$P(\theta \mid D)$





Confidence



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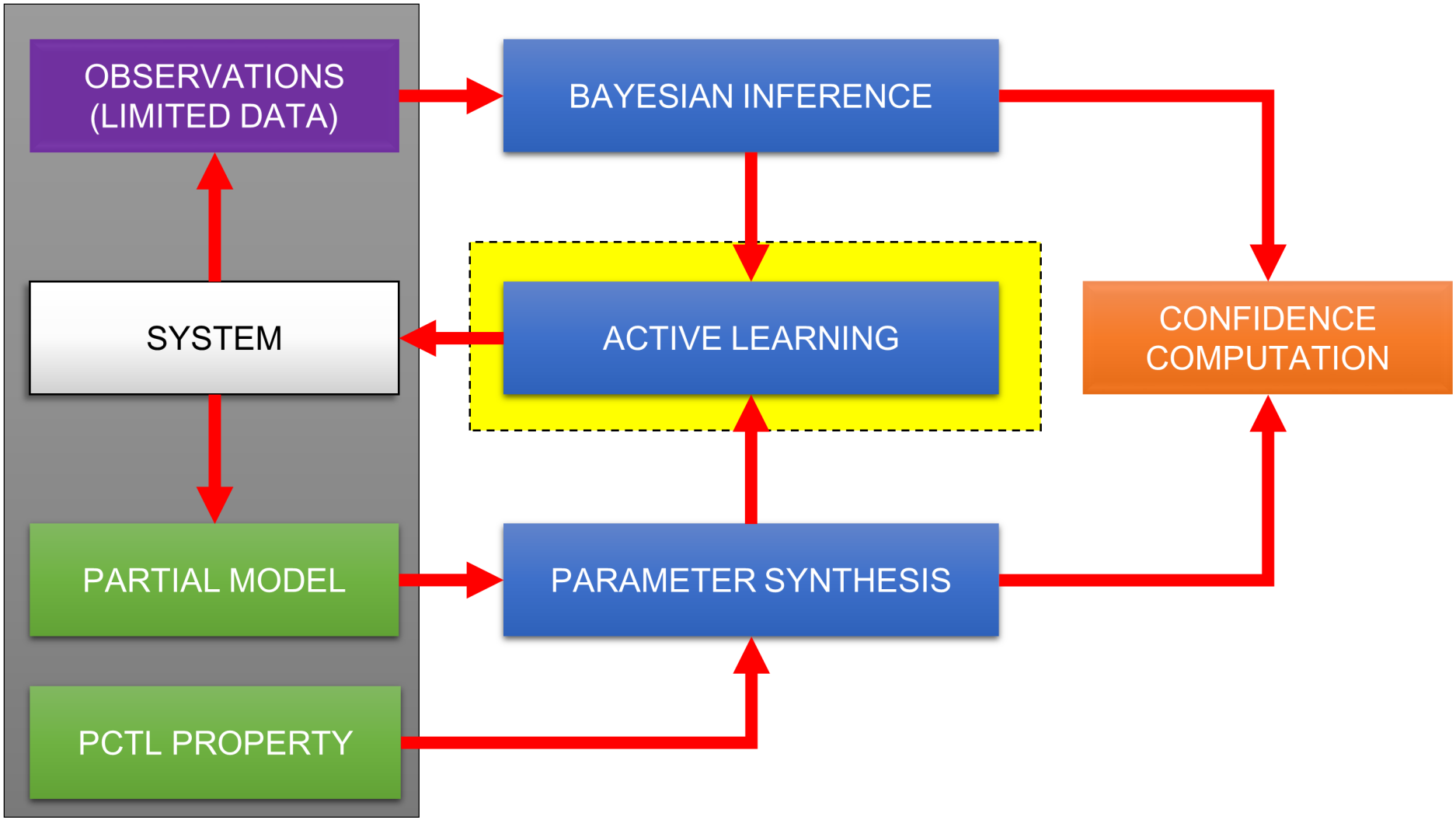
Overview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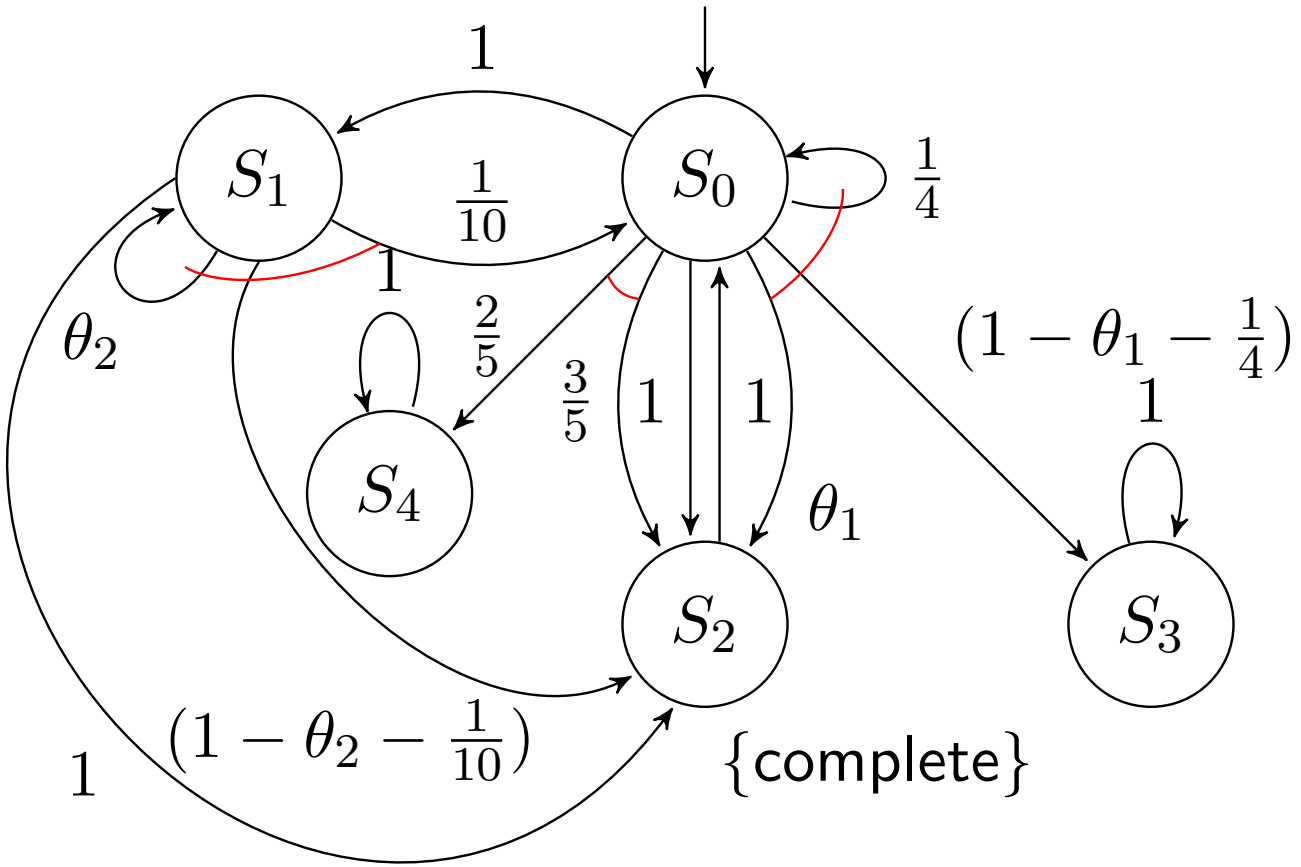
Data-Efficiency

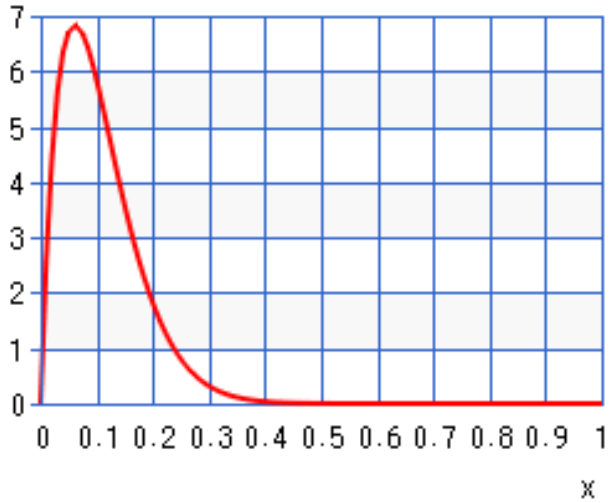
Some data tells us more about whether the property is satisfied



$\mathbf{P}_{\geq 0.5}(\text{true } \mathcal{U} \text{ complete})$

Role of Actions in Data Acquisition

Actions can affect posterior distribution $P(\theta \mid \mathscr{D})$



Actions can affect confidence $\mathscr{C}$

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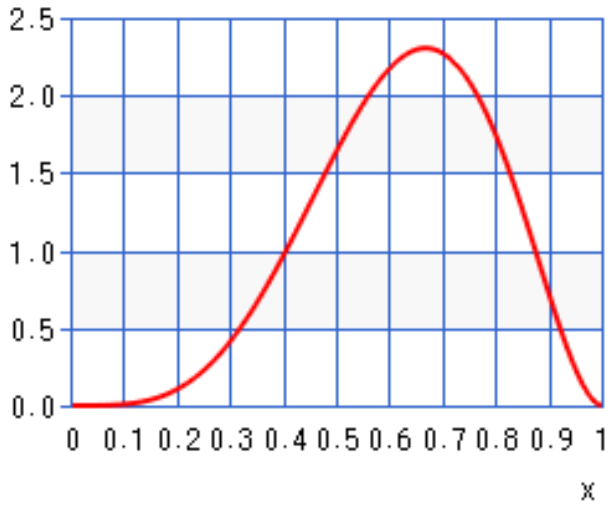

Actions can affect confidence $\mathscr{C}$

Information State Space

Employs the joint space of states and information

Information State (Memory) $\mathbf{m} = s_0 a_0 s_1 a_1 s_2 a_2 \ldots$

Belief State $\hat{s} = (s, \mathbf{m})$

$\hat{s}$ is a statistic of the memory: $\hat{s} = f(\mathbf{m})$ and summarises all information accumulated so far

$$P(\theta_i \mid \mathscr{D}) = Beta(\theta_i; \bar{C}(\theta_i) + \omega) \propto \theta_i^{C(\theta_i)} (1 - \theta_i)^{C'(\theta_i)}$$

$$\mathbf{m} \leftarrow \mathrm{i}_{\mathbf{m}}$$

Joint Beta distribution with hyperparameters $(\boldsymbol{\alpha}, \boldsymbol{\beta})$

$$\boldsymbol{\alpha} = \langle C(\theta_1) + \alpha_{\theta_1}, C(\theta_2) + \alpha_{\theta_2}, \ldots, C(\theta_n) + \alpha_{\theta_n} \rangle$$

$$\boldsymbol{\beta} = \langle C'(\theta_1) + \beta_{\theta_1}, C'(\theta_2) + \beta_{\theta_2}, \ldots, C'(\theta_n) + \beta_{\theta_n} \rangle$$

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$$\mathbf{m} \leftarrow \mathbf{i_m}$$

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$$(s, \mathbf{m}) \leftarrow (s, \mathbf{i_m})$$

Bayes-Adaptive MDP (BAMDP)

belief states — finite set of actions — $(s, \mathbf{m}) \in \hat{\iota}$ if $s \in \iota$

$$\mathcal{M}_{ba} = (\hat{S}, A, \hat{\mathcal{T}}, \hat{\iota}, \mathcal{R})$$

Extra information is the posterior distribution over the parameters

$$P(\theta \mid \mathscr{D}) = b(\mathcal{T}_p) = P(\mathcal{T}_p \mid \mathrm{i}_{\mathbf{m}}) \propto P(\mathrm{i}_{\mathbf{m}} \mid \mathcal{T}_p) P(\mathrm{i}_{\mathbf{m}})$$

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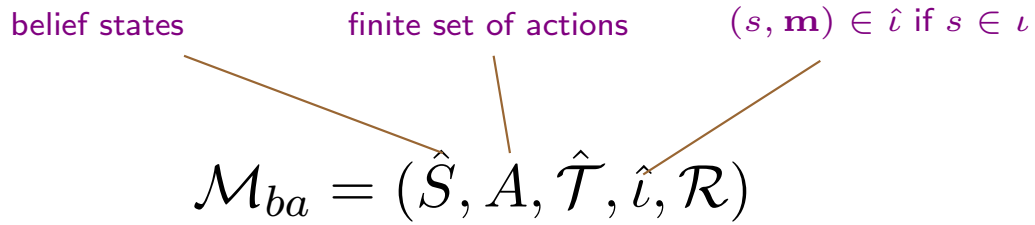
Bayes-Adaptive MDP (BAMDP)



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belief states — finite set of actions — $(s, \mathbf{m}) \in \hat{\iota}$ if $s \in \iota$

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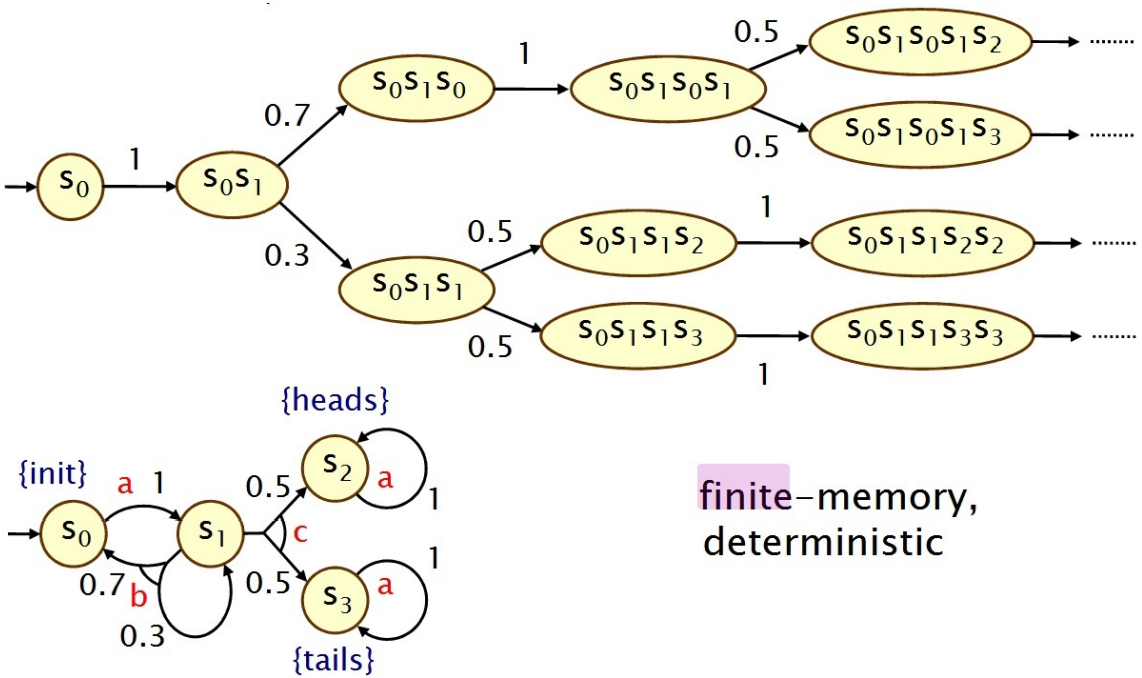
Classification of Strategies

Memoryless strategy: $a_t = \pi(s_t), \forall t > 0$

Finite-memory strategy: There are finitely many modes M s.t. $a_t = \hat{\pi}(s_t, m_t), \forall t > 0$ where $m_t \in M$ is the current mode



Choose b then c in s_1

Figure courtesy of Marta Kwiatkowska

Reward Function

Data-Efficiency: want to maximise the information content of the observations

Need immediate rewards:- **immediate confidence gain**:

$$r_{t+1} = |0.5 - \mathscr{C}_{t+1}| - |0.5 - \mathscr{C}_t|$$

Want to maximise gain:- design reward function for BAMDP:

$$\mathcal{R}(s, \mathbf{i_m}, a) = \mathbb{E}[r_{t+1} \mid s_t = s, \mathbf{i}_{\mathbf{m}_t} = \mathbf{i_m}, a_t = a]$$

Q function: $Q^{\hat{\pi}}(s, \mathbf{m}, a) = \mathbb{E}_{\hat{\pi}}[\Sigma_{j=t}^{T} r_j \mid s_t = s, \mathbf{m}_t = \mathbf{m}, a_t = a]$

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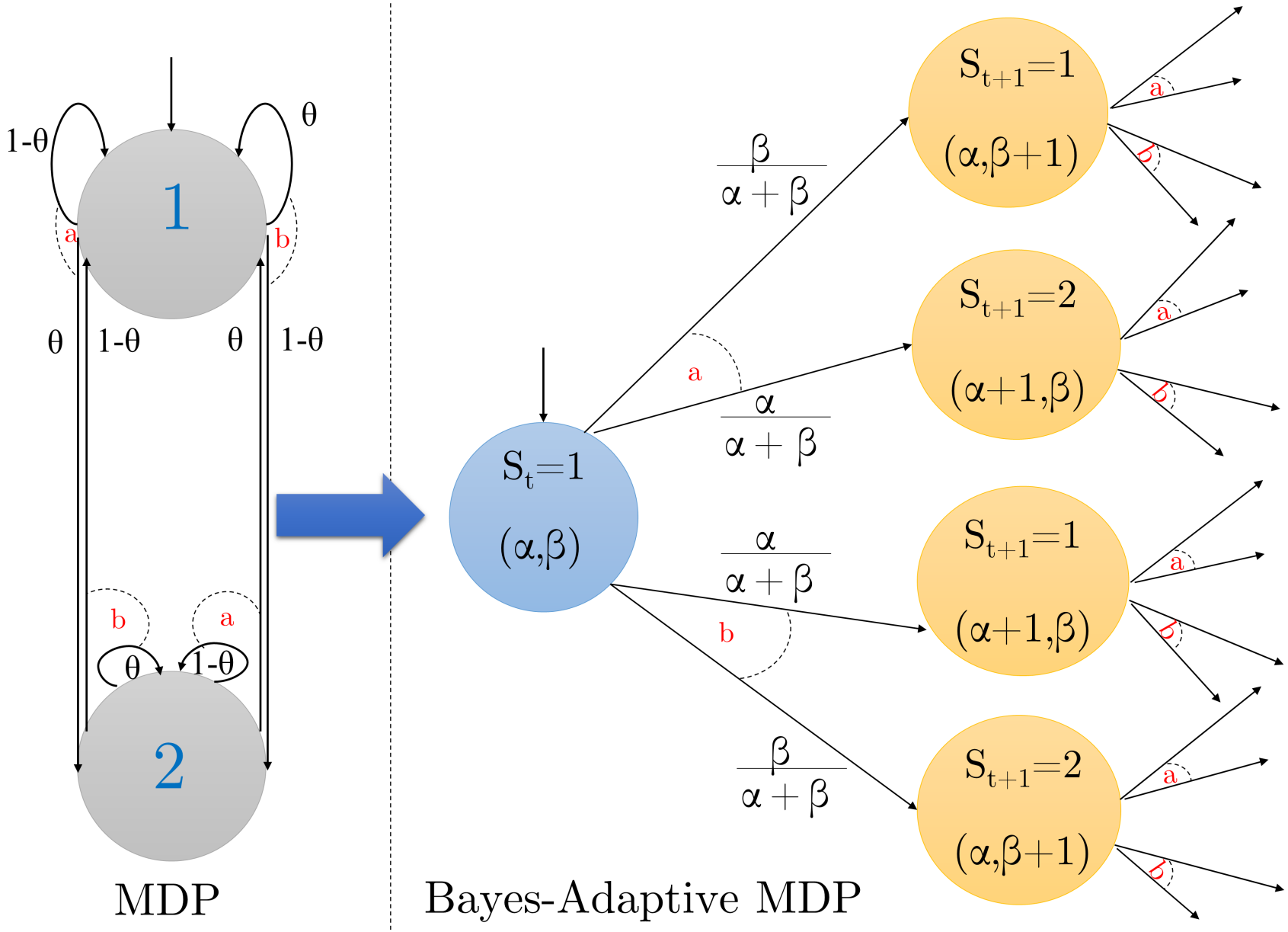
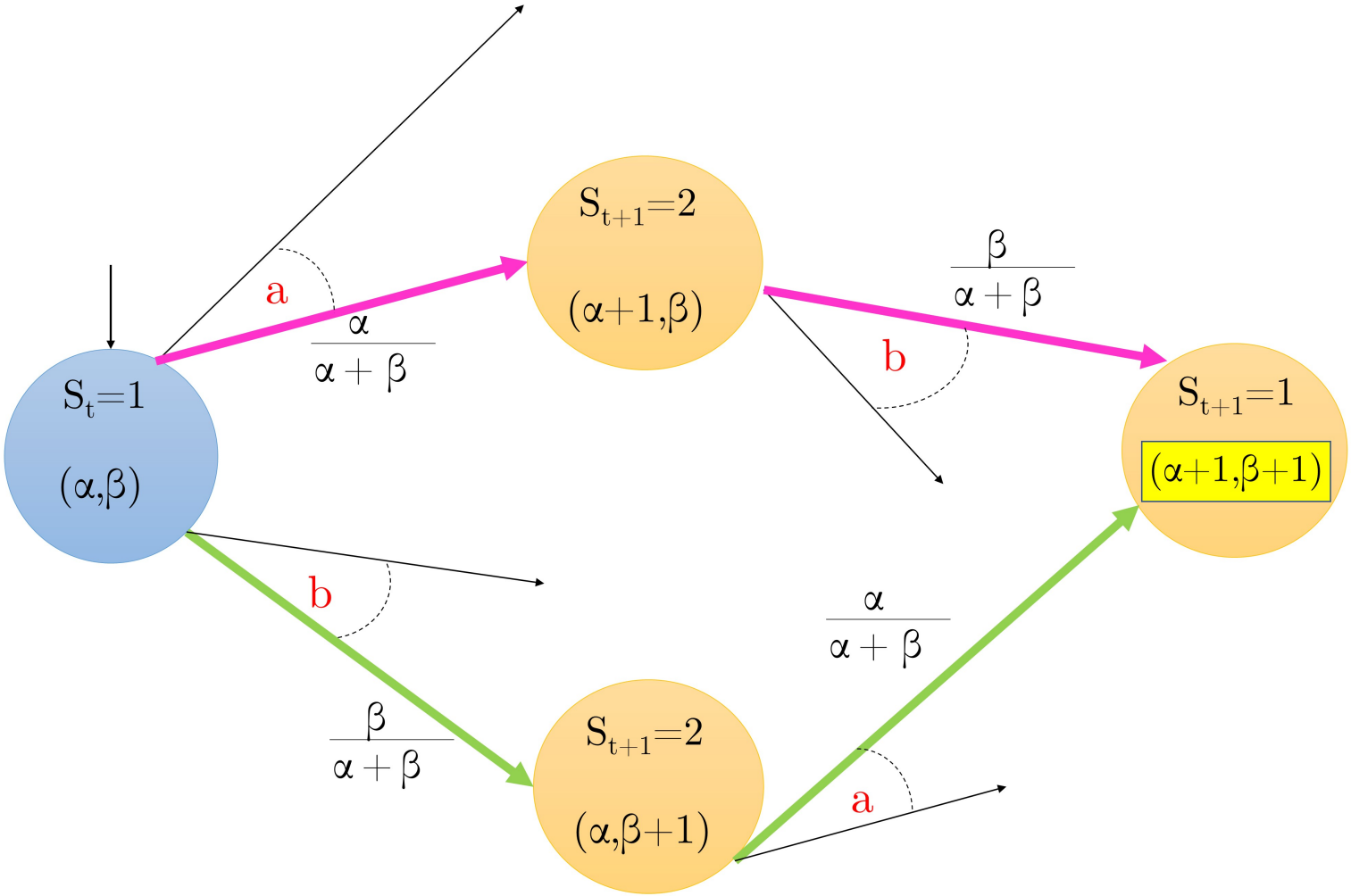
Bayes-Adaptive MDPs are Sizeable



Value Function Approximation

Different memories can correspond to the same posterior belief



Generalise from seen to unseen states

Outline

Approximate Bilinear Q Function

$$Q(s, \mathbf{i_m}, a; \beta) = \mathbf{y}(\mathbf{i_m})^\top \beta \mathbf{x}(s, a)$$

- Goal of the learning process is to find β that minimises $\mathbb{E}[(Q(s, \mathbf{i_m}, a; \beta) - Q(s, \mathbf{i_m}, a))^2]$
- $\mathbf{x}(s, a)$ is a domain-dependent state-action feature vector: we use one-hot encoding to represent $\mathbf{x}(s, a)$
- Construct $\mathbf{y}\colon \widehat{\mathbf{M}} \to \mathbb{R}^{\mathcal{Z}}$ with $\mathcal{Z}$ as the degree of finite dimensional approximation and $\mathbf{i_m} \in \widehat{\mathbf{M}}$
- Update $\mathbf{y}_j(\mathbf{i_{mas'}}) = \mathbf{y}_j(\mathbf{i_m})\hat{\mathcal{T}}(s, \mathbf{i_m}, a, s', \mathbf{i_{mas'}})$ allows generalisation:

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Bayes-Adaptive Temporal Difference Search

Inputs:
  s_t, $\mathbf{i}_{\mathbf{m}_t}$
Initialize:
  $\beta \leftarrow 0$, $\rho \leftarrow 0$
procedure SEARCH(s_t, $\mathbf{i}_{\mathbf{m}_t}$)
  while time remaining **do** ▷ Start episode
    $s \leftarrow s_t$, $\mathbf{i}_{\mathbf{m}} \leftarrow \mathbf{i}_{\mathbf{m}_t}$, $\tilde{t} \leftarrow t$
    $a \leftarrow \hat{\pi}_{\epsilon\text{–greedy}}(s, \mathbf{i}_{\mathbf{m}}; Q)$
    $\xi \leftarrow 0$, $\xi' \leftarrow 0$
    while $\tilde{t} < T$ **do**
      $s' \sim \hat{\mathcal{T}}(s, \mathbf{i}_{\mathbf{m}}, a, \cdot, \mathbf{i}_{\mathbf{ma}\cdot})$
      $R \leftarrow \mathcal{R}(s, \mathbf{i}_{\mathbf{m}}, a)$
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      $\beta \leftarrow \beta + \alpha\delta\xi - \alpha\xi'(\xi^\top\rho)$
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      $s' \sim \hat{\mathcal{T}}(s, \mathbf{i}_{\mathbf{m}}, a, \cdot, \mathbf{i}_{\mathbf{m}a\cdot})$
      $R \leftarrow \mathcal{R}(s, \mathbf{i}_{\mathbf{m}}, a)$
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  return $\arg\max_{a_t} Q(s_t, \mathbf{i}_{\mathbf{m}_t}, a_t; \beta)$
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Inputs:
  s_t, $\mathbf{i}_{\mathbf{m}_t}$
Initialize:
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procedure SEARCH(s_t, $\mathbf{i}_{\mathbf{m}_t}$)
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    $s \leftarrow s_t$, $\mathbf{i}_{\mathbf{m}} \leftarrow \mathbf{i}_{\mathbf{m}_t}$, $\tilde{t} \leftarrow t$
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    end while
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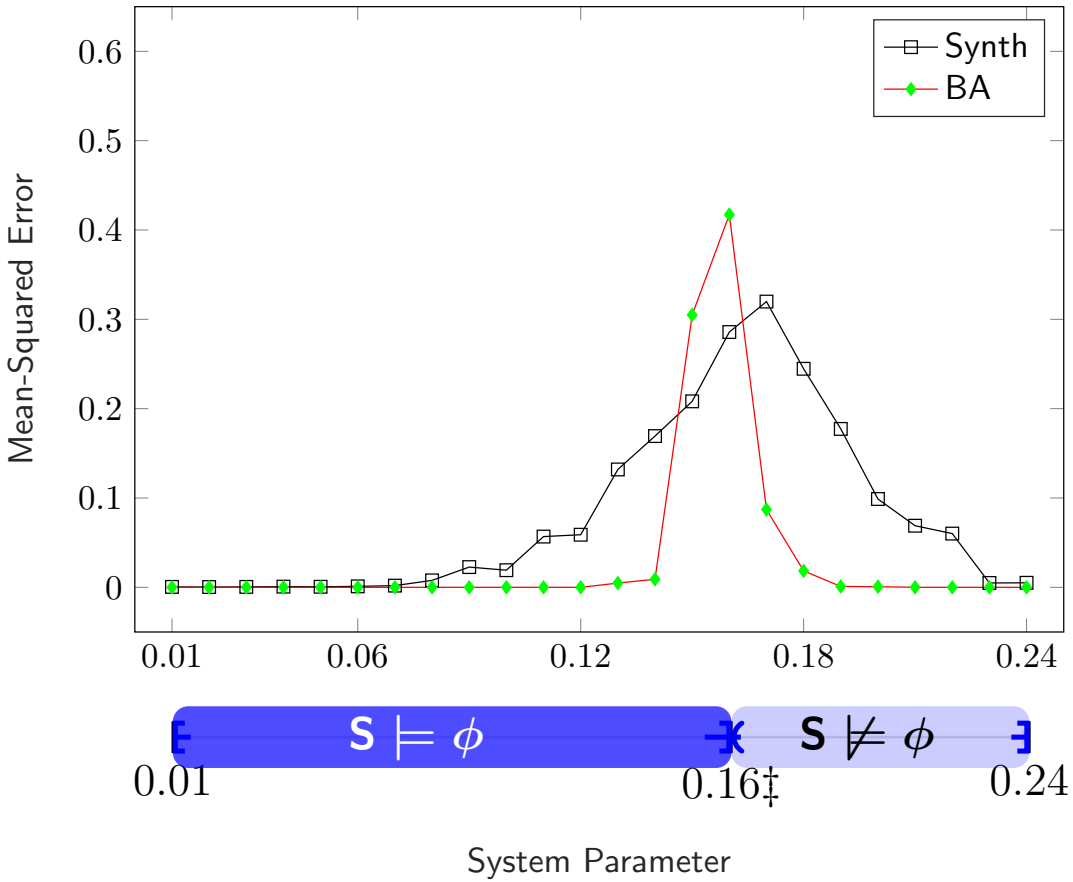
  return $\arg\max_{a_t} Q(s_t, \mathrm{i}_{\mathbf{m}_t}, a_t; \beta)$
end procedure

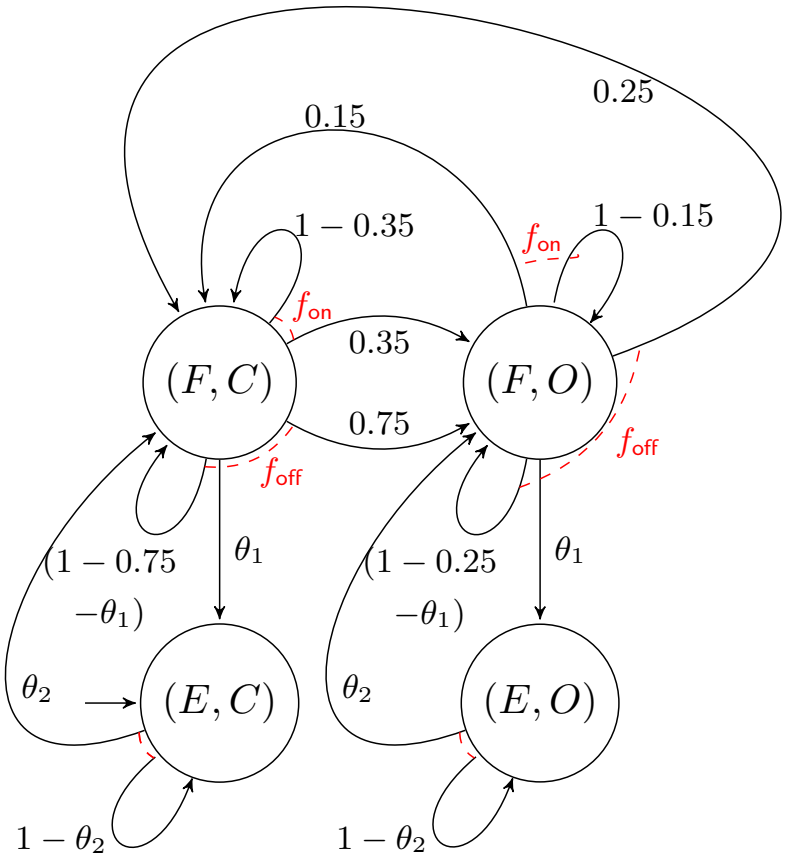
Outline

Evaluation - Case 2

$$\mathbf{P}_{\geq 0.35} \neg (\text{true } \mathcal{U}^{\leq 20} \ (\mathsf{E}, \mathsf{O}))$$

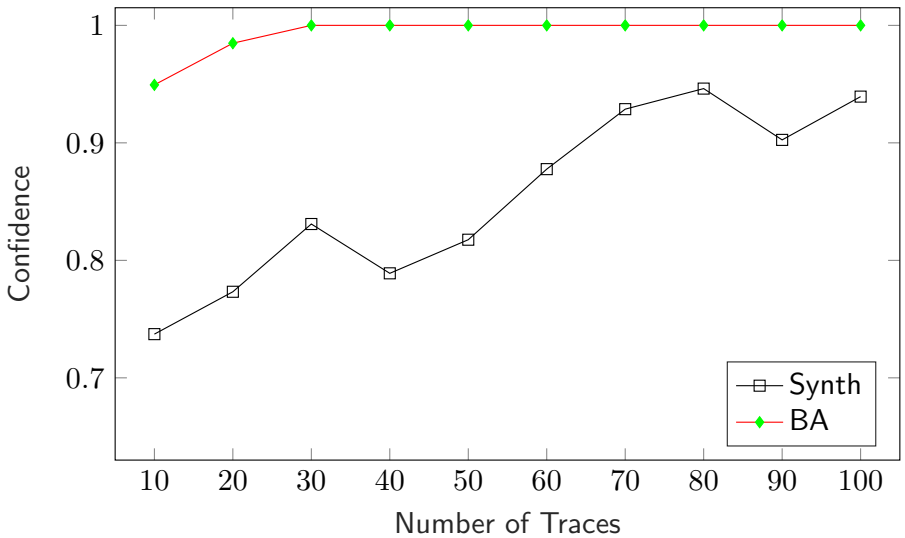
System Parameter $\theta_* = (\theta_1 = \theta_2)$

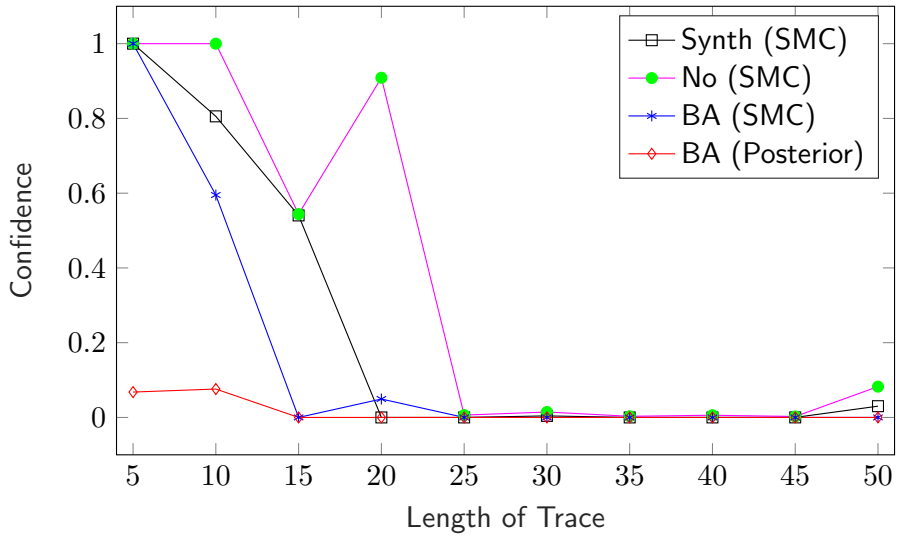




Evaluation - Case 2

$$\mathbf{P}_{\geq 0.35} \neg (\text{true } \mathcal{U}^{\leq 20} \text{ (E, O)})$$

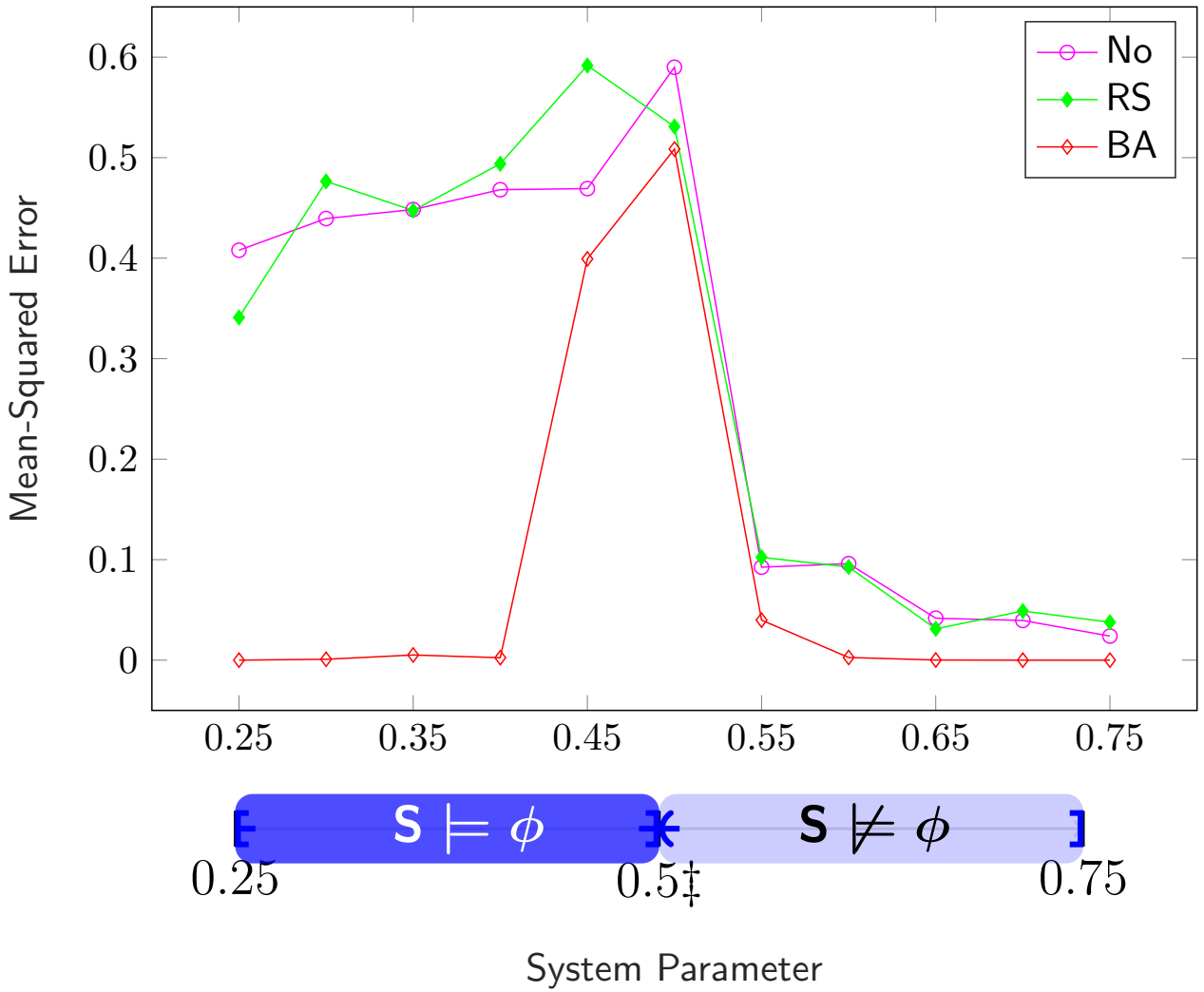




Evaluation - Case 3

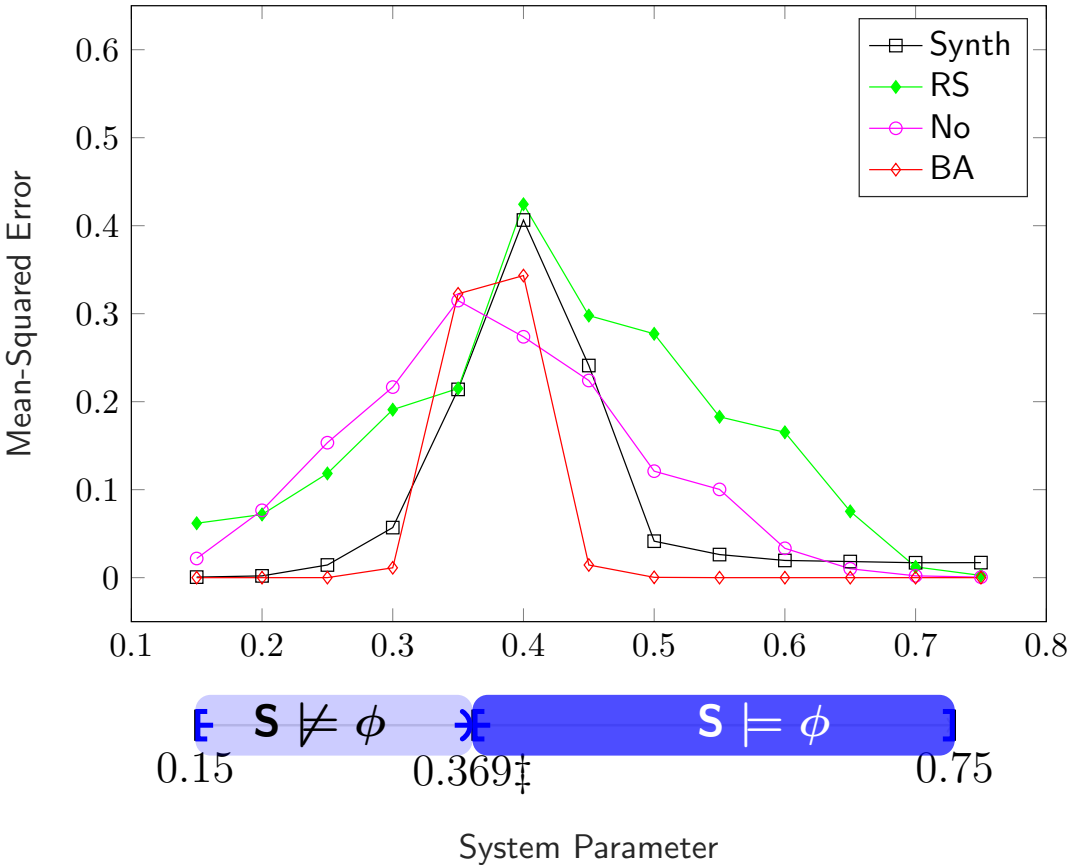
$\mathbf{P}_{\geq 0.25}(\text{true } \mathcal{U} \text{ (finished } \& \text{ allCoinsEqualToOne))}$

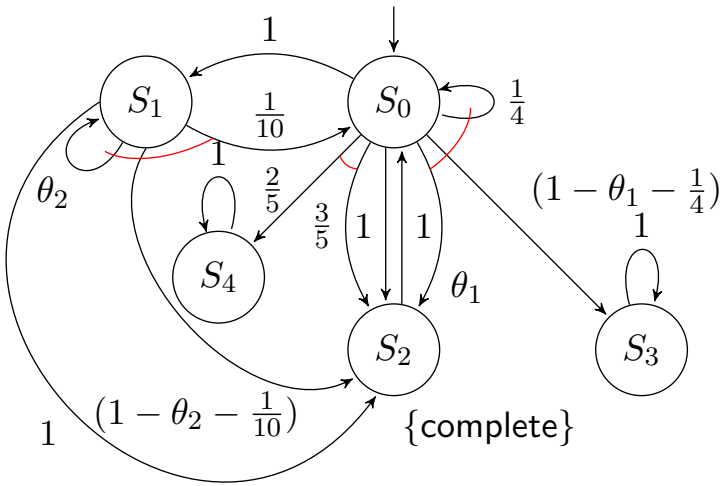
4112 states 7692 transitions



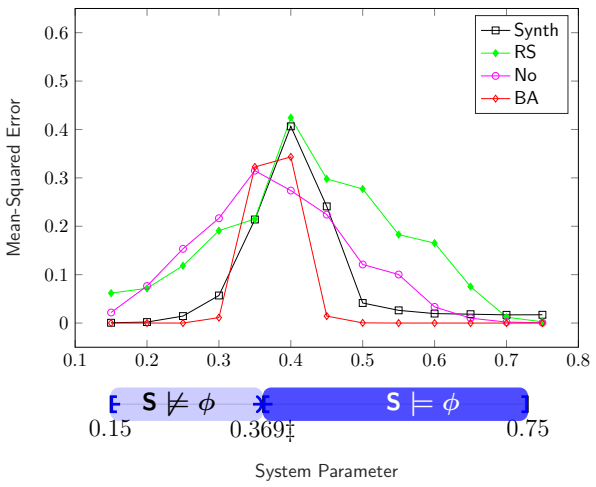
Evaluation - Case 1

$$\mathbf{P}_{\geq 0.5}(\text{true } \mathcal{U} \text{ complete})$$

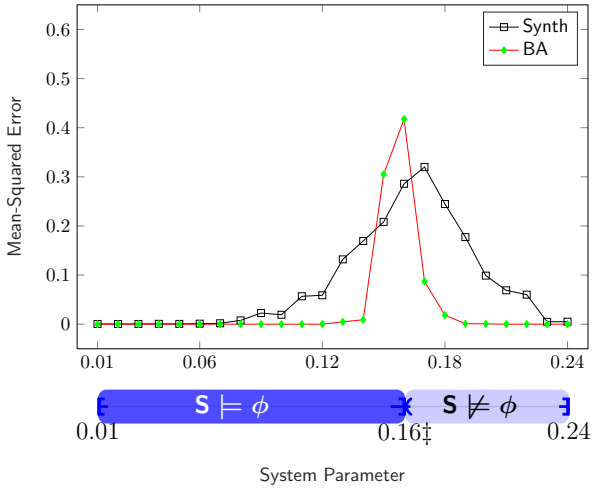




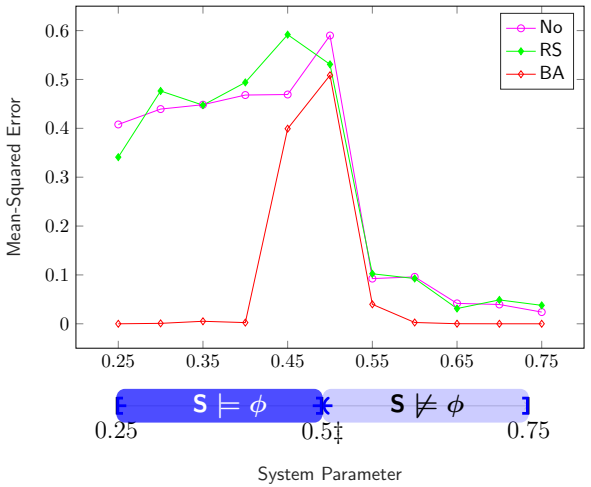
Evaluation - Summary



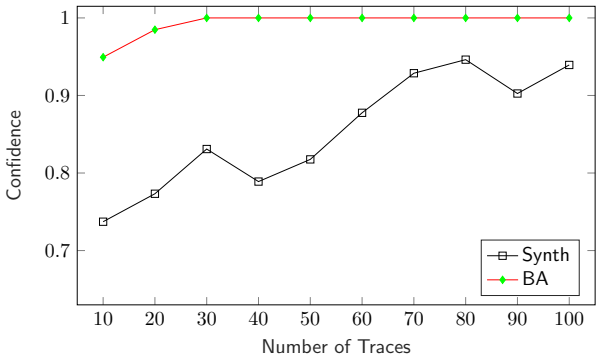
(a) MSE for Case 1 ($t10$,$l10$)



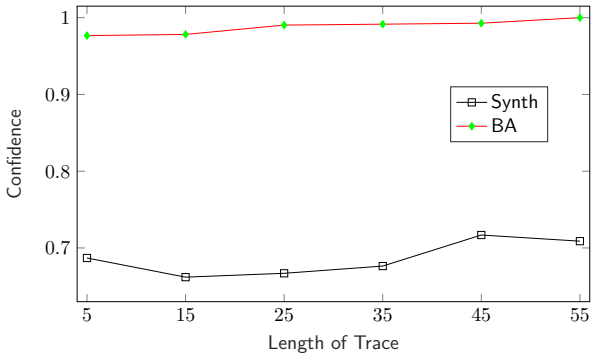
(b) MSE for Case 2 ($t10$,$l20$)



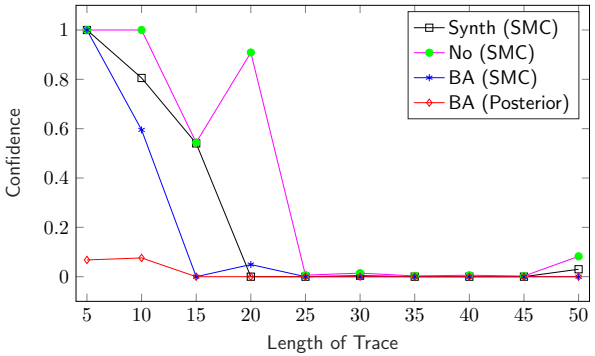
(c) MSE for Case 3 ($t10$,$l10$)



(d) Case 2 ($\theta_* = 0.13$,$l20$)



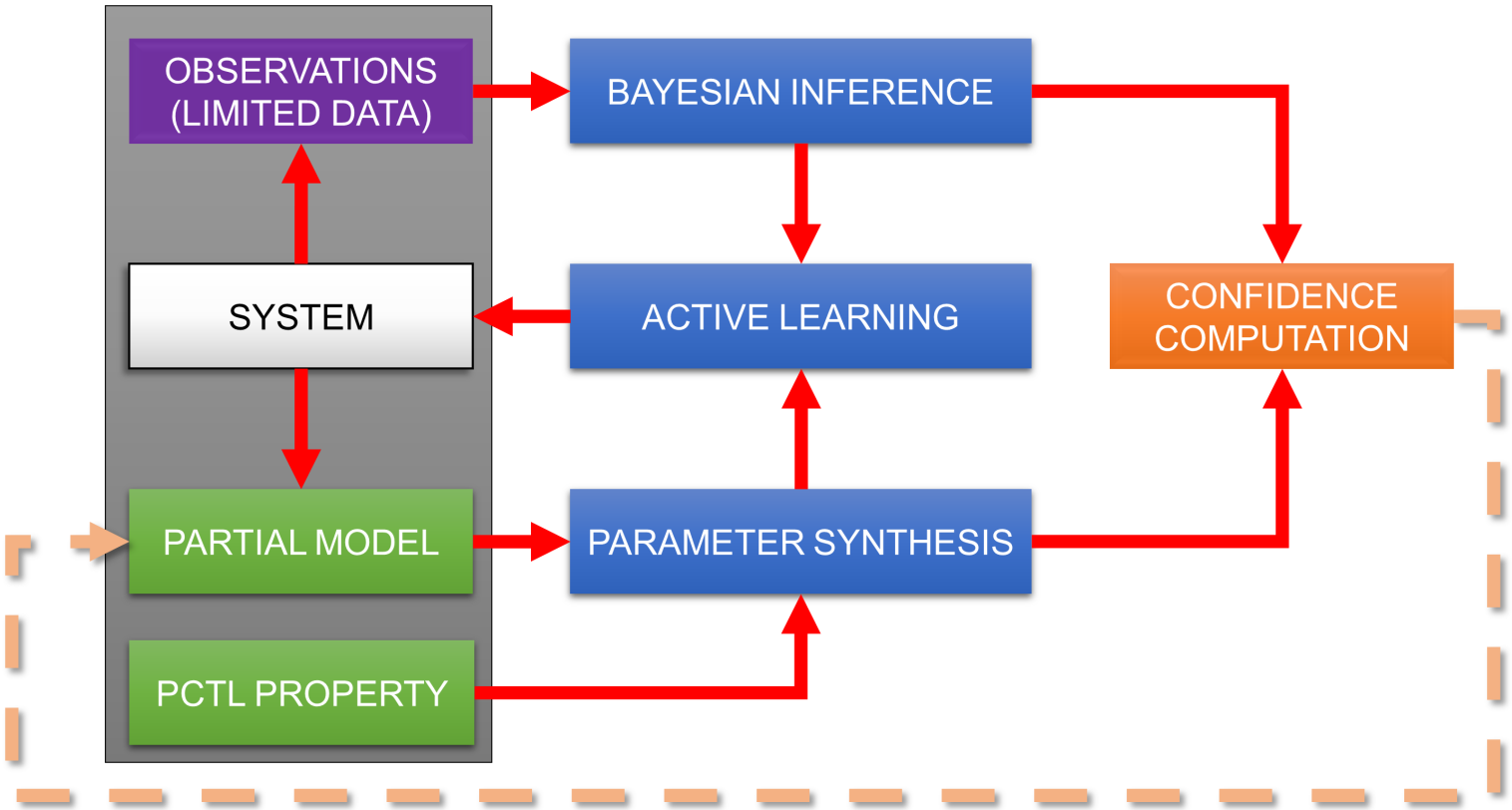
(e) Case 2 ($\theta_* = 0.14$,$t10$)



(f) Case 2 ($\theta_* = 0.24$,$t10$)

Takeaway Message

Verification of physical systems: Integrated learning and verification



Epilogue

Conclusions

- Our approach is able to gather more useful data
- Robust to the amount of data gathered
- Can work with much shorter horizons
- Can handle large MDP models
- Takes into account memory when calculating optimal strategies
- Actions are exclusively selected for learning tasks

Future Work

- Tune/repair model until a decisive confidence is achieved
- Actions can affect model classification as well

OXFORD CONTROL AND VERIFICATION GROUP (OXCAV)

Visit our group page: http://www.oxcav.org

PI: Prof. Alessandro Abate

Questions?

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E. Polgreen, V. B. Wijesuriya, S. Haesaert, and A. Abate, "Data-efficient bayesian verification of parametric markov chains," in *Quantitative Evaluation of Systems* (G. Agha and B. Van Houdt, eds.), (Cham), pp. 35–51, Springer International Publishing, 2016.


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